

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 1916

THE EFFECTS OF SIDESLIP, ASPECT RATIO, AND MACH NUMBER
ON THE LIFT AND PITCHING MOMENT OF TRIANGULAR,
TRAPEZOIDAL, AND RELATED PLAN FORMS IN
SUPERSONIC FLOW

By Arthur L. Jones, Robert M. Sorenson,
and Elizabeth E. Lindler

Ames Aeronautical Laboratory
Moffett Field, Calif.

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THE EFFECTS OF SIDESLIP, ASPECT RATIO, AND MACH NUMBER ON THE
LIFT AND PITCHING MOMENT OF TRIANGULAR, TRAPEZOIDAL,
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SUMMARY

The variations of lift and pitching moment with sideslip in supersonic flow have been calculated for a representative group of plan forms. The variations of lift and pitching moment with aspect ratio and Mach number at zero sideslip were also investigated. The analysis was based on linearized potential theory and was applied to triangular, trapezoidal, rectangular, and swept-back plan forms without dihedral.

In general, the lift and pitching moment showed a slight but steady increase in magnitude as the sideslip angle was increased. For a fixed Mach number, the only decrease in lift or in the magnitude of the pitching moment with increasing aspect ratio was the decrease in the latter parameter for the trapezoidal plan forms with raked-out tips. With the aspect ratio fixed and Mach number increasing, the only increases in either of these parameters was in the magnitude of the pitching moment for the swept-back plan forms.

INTRODUCTION

Although a knowledge of the variations of lift and pitching moment with sideslip is not of great value in dynamic stability calculations, inasmuch as these variations provide a cross derivative which does not fit into either the longitudinal or lateral equations of motion, the knowledge of these variations is of interest in the analysis of the static stability and the flying qualities of an airplane. This information is useful also in the study of controlled maneuvers and the step-by-step calculations of airplane motion for

large sideslip angles. Expressions for the variation of lift with sideslip were, in a large part, available from a previous investigation (reference 1) to determine the rolling moment due to sideslip. These expressions could be supplemented readily with both the lift and pitching-moment expressions needed to complete the analysis. Consequently, the material was assembled to provide some desirable additional data on the theoretical aerodynamic characteristics of supersonic wing plan forms. Expressions are included for the lift and pitching moment at zero sideslip as functions of aspect ratio and Mach number.

The plan forms investigated herein are shown in figures 1 and 2 and fit the following descriptions: (1) triangular with subsonic leading edges or with supersonic leading edges; (2) trapezoidal with all possible combinations of raked-in, raked-out, subsonic, or supersonic tips; (3) rectangular; and (4) two swept-back plan forms with supersonic trailing edges developed from the triangular wings.

Previously published reports that contained data on lift and pitching-moment variations with sideslip in supersonic flow are reference 2 for triangular plan forms and reference 3 for thin pointed wings with several types of trailing edges.

SYMBOLS AND COEFFICIENTS

A aspect ratio $\left(\frac{b^2}{S} \right)$

b span of wing measured normal to plane of symmetry

B $\sqrt{M_1^2 - 1}$

Bm ratio of tangent of right tip angle to tangent of Mach cone
angle $\left(\frac{m}{\tan \mu} \right)$

c chord of wing measured parallel to plane of symmetry

c_r chord of wing root

\bar{c} mean aerodynamic chord $\left(\frac{\int c^2 dy}{\int c dy} \right)$

C_L lift coefficient $\left(\frac{L}{qS} \right)$

C_m pitching-moment coefficient $\left(\frac{M}{qSc} \right)$

$E(\phi, k)$ incomplete elliptic integral of the second kind with modulus k

$$\left(\int_0^\phi \sqrt{1-k^2 \sin^2 \theta} d\theta \right)$$

E complete elliptic integral of the second kind with modulus k

$$\left[E \left(\frac{\pi}{2}, k \right) \right]$$

$F(\phi, k)$ incomplete elliptic integral of the first kind with modulus k

$$\left(\int_0^\phi \frac{d\theta}{\sqrt{1-k^2 \sin^2 \theta}} \right)$$

K complete elliptic integral of the first kind with modulus k

$$\left[F \left(\frac{\pi}{2}, k \right) \right]$$

l over-all longitudinal length of swept-back wing

L lift

m slope of right wing tip relative to plane of symmetry in
plane of the wing
(Positive for raked-out tip, negative for raked-in tip.)

M pitching moment about lateral body axis through the leading
edge of root chord
(Diving moments are negative.)

M_1 free-stream Mach number

ΔP pressure differential across wing surface
(Positive upward.)

q free-stream dynamic pressure $\left(\frac{\rho}{2} V^2 \right)$

S area of wing

V	free-stream velocity
x,y	rectangular coordinates of wind axes
α	angle of attack, radians
β	sideslip angle, degrees (Positive when sideslipping to right)
μ	Mach angle $\left(\tan^{-1} \frac{V}{B} \right)$
ξ, η	rectangular coordinates of body axes
$\bar{\xi}$	longitudinal center-of-lift location in the body axes coordinates
ρ	air density in the free stream

Subscripts

A,B,C, . . . V expressions for the lift and pitching moment presented in Appendix B.

METHOD OF ANALYSIS

The distributions of the normal force on the plan forms in sideslip were calculated originally during the investigation to determine the rolling moment due to sideslip reported in reference 1. The method used to calculate the normal-force distribution amounted to an application of the supersonic lifting-surface theories presented in references 4 and 5. On an actual wing it is seldom that the tip shape very closely approximates the sharp-edged condition that the thin-airfoil restriction of the lifting-surface theories implies, and it is possible that a well-rounded edge raked at a very small angle away from the free stream might support a flow that would not yield zero pressure difference at the edge. (See reference 6.) For a theoretical analysis, however, it is not yet possible to specify the precise flow or pressure condition that will exist at the edge, nor is it possible to specify the manner in which the transition from any assumed flow or pressure condition to the Kutta condition will take place as the sideslip angle is increased. Thus, it is most practical to base an analysis for finite values of the sideslip angle on the Kutta condition.

The integration of the normal forces to obtain expressions for the lift on portions of certain plan forms was completed in reference 1, in order to assist in the transformation of rolling moments from localized body axes to the body axes located in the plane of symmetry. The remainder of the lift expressions were obtained by a simple integration of the normal forces over the wing plan form.

The pitching moments were evaluated by the following integration:

$$M = \iint_{\text{plan form}} \Delta P \xi d\xi d\eta$$

where $\Delta P d\xi d\eta$ is the normal force and ξ is the moment arm measured from a lateral axis through the leading edge of the root chord; that is, ξ is the longitudinal dimension in the system of body axes having its origin at the leading edge of the root chord.

The pitching moments calculated in the manner indicated can be referred to either the body axes or the stability axes. A rigorous calculation of the lift could be obtained by determining the normal force and reducing it by $\cos \alpha$; however, inasmuch as the angle of attack permissible is small for an analysis based on linearized theory, the factor $\cos \alpha$ was neglected.

As explained in more detail in the presentation of the results, it was found to be convenient to break down the plan forms into sectors in order to simplify the analysis and to simplify the presentation of the resulting expressions. The changes in configuration between the Mach cone traces and the wing edges that a wing in sideslip undergoes suggested the form for these simplifications. As the tips change from subsonic to supersonic or vice versa, and as the edges and tips change figuratively from leading to trailing edges by swinging past the free-stream direction, the load distribution is altered considerably. Consequently, it was not only desirable to divide the plan form into sectors, but it was also desirable to divide the sideslip rotation into a number of phases, in order that the expressions for the lift and pitching moment could be provided for each configuration encountered in the range of sideslip investigated.

The plan forms are classified with regard to the relative positions of the wing tips and tip Mach cones when the wing is at zero sideslip. The ratio of the tangent of the right tip angle to the tangent of the Mach cone angle B_m makes a convenient

index for classification. The slope of the right tip m is defined as positive when the tip is raked out and negative when the tip is raked in. If Bm is equal to or greater than 1, the tips are supersonic leading edges. If Bm is equal to or less than -1, the tips are supersonic trailing edges. For values of Bm between 1 and -1, the tips are subsonic.

RESULTS AND DISCUSSION

For most of the plan forms investigated, the complete expressions for the lift and pitching moments are rather long and involved. Certain plan-form sectors contribute a part to these expressions that is applicable to more than one plan form and in more than one of the phases of sideslip. Rather than writing in full the expressions for each plan form in each phase of sideslip, it was decided to eliminate needless repetition by symbolizing the moment and lift expressions contributed by various sectors of the plan forms or by whole plan forms and presenting the formulas for lift and pitching moment in Appendix A in terms of these symbols. Appendix B gives the expressions associated with the symbols. Thus Appendixes A and B contain the general results of this investigation. These results in their involved form are not particularly susceptible to a tangible physical interpretation. In order to make a closer study of these results, calculations of the lift and pitching-moment variations with sideslip were made for a number of typical plan forms. Two values of the Mach number parameter B , 1 and 1.5, were used in these calculations and the data obtained are presented in graphical form in figures 3 through 6.

Examination of these data indicated that the effects of sideslip were rather small and insignificant. In order to supplement these data, therefore, it was decided to investigate the variations with aspect ratio and with Mach number of lift and pitching moment at zero sideslip. Such material is already available to a certain extent but from sources that are fairly well scattered (see reference 7 for the most complete compilation heretofore) and it appeared to be advantageous to include in one report the expressions for the lifts and pitching moments of these plan forms at zero sideslip. Appendix C contains these expressions. It was possible to present these expressions in a completely assembled form, due to the fact that the restriction to zero sideslip results in a great simplification in the unassembled expressions of Appendix B. The graphical data corresponding to these expressions are shown in figures 7 and 8.

Variation of Lift and Pitching Moment with Sideslip

The variations of C_L/α and C_m/α with sideslip, shown in figures 3 through 6, are practically negligible for a reasonably large range of sideslip angles. For the trapezoidal and rectangular plan forms the variation is, without exception, a slight but steady increase in lift or in the magnitude of the pitching moment as the sideslip angle is increased. The triangular plan forms showed the same slight increase until an edge passed through a Mach cone from the apex or tip. When such a passage occurred, the curve made a definite break and reversed in slope. The resulting decrease in magnitude of the lift or pitching moment was more pronounced than the preceding increase. The analysis of the base-forward triangular plan forms was restricted to plan forms having supersonic trailing edges by practical limitations of the linearized theory applied. For this type of plan form, the variations of lift and pitching moment with sideslip were similar to the variations for the triangular plan forms. Due to the complexity of the analysis involved, the swept-back plan forms were not analyzed beyond the point where the leading edge coincided with the apex Mach cone. Up to this point, the lift and pitching moment underwent the usual slight but steady increase with increasing sideslip. It is probable that the swept-back plan forms would follow the variations of the closely related triangular plan forms throughout the entire range of sideslip angles.

The changes in aspect ratio and in the Mach number parameter B that were included in these calculations did not seem to have any appreciable effect on the variations of C_L and C_m with sideslip.

In general, the graphical results showed that sideslip does not destroy the property of fore-and-aft reversibility of the plan forms (reference 8) as far as the lift is concerned. With regard to the pitching moment, however, the plan forms do not show any reversibility property whatsoever.

Lift and Pitching Moment at Zero Sideslip

Effects of plan form.— With regard to the relative magnitudes of the lift and pitching-moment parameters for the various plan forms, the values for the triangular plan form with the apex forward can be used as a good basis for comparison. This deduction follows from inspection of the curves in figures 7 and 8 which show that the values of BC_L/α and BC_m/α for the triangular plan form are fairly

representative of the mean values of these parameters for the group of plan forms investigated. Furthermore, the curves for BC_L/α and BC_m/α are identical for the triangular plan form. In the range from $BA=0$ to $BA=4$, the triangular plan forms have subsonic leading edges and the values of BC_L/α and BC_m/α increase fairly steadily from 0 to 4. For values of BA greater than 4, the triangles have supersonic leading edges and BC_L/α and BC_m/α remain constant at 4.

The base-forward triangular plan forms with supersonic trailing edges have the same lift as the triangular plan forms with the apex forward but only half the pitching moment.

The magnitudes of BC_L/α and BC_m/α for the swept-back wings were generally greater than those for the triangular wings; whereas the trapezoidal and rectangular plan forms, in general, possessed smaller magnitudes of these parameters than the triangular wings. At the lower values of BA the subsonic-edged swept-back wings and the trapezoidal and rectangular wings provided exceptions to the general rule for the magnitudes of the lift parameter.

Effects of aspect ratio.— With the Mach number fixed, the variations of C_L/α and C_m/α with aspect ratio become equivalent, qualitatively, to the variations of BC_L/α and BC_m/α with BA. Thus, by direct inspection of figures 7 and 8, it can be seen that, with the exception of some of the trapezoidal plan forms with raked-out tips, the magnitudes of both C_L/α and C_m/α increase with increasing aspect ratio. For the trapezoidal plan forms that proved to be the exceptions, the pitching-moment parameter decreases with increasing aspect ratio.

Effects of Mach number.— With the aspect ratio fixed and the Mach number parameter B varying, the qualitative interpretation of the curves in figures 7 and 8 cannot be obtained by a simple inspection of the curves. A rule of thumb exists, however, that can be applied to simplify the interpretation: To determine whether there is an increase or decrease in the magnitude of either the lift or pitching-moment parameter as B increases, consider the slope of the curve at the point in question and the slope of a radial line from the origin to the point. If the slope of the tangent is less than the slope of the radial line, the magnitude of the parameter decreases with increasing B. If the slope of the tangent is greater than the slope of the radial line, the magnitude of the parameter increases with increasing B.

Thus it can be determined that, for most of the plan forms, C_L/α and C_m/α decrease in magnitude as B increases. The magnitude of C_m/α for the supersonic-edged swept-back plan forms, however, increases without exception as B is increased and the swept-back plan forms with subsonic leading edges are marginal in this respect.

CONCLUDING REMARKS

The variations of lift and pitching moment with sideslip calculated for selected typical plan forms were found to be rather small. The usual pattern was a slight but steady increase in the magnitudes of C_L and C_m as the sideslip angle was increased. The only deviations from this pattern were observed for the triangular plan forms. At the sideslip angle where the edge of a triangular plan form passed through the Mach cone from the apex or tip, the curve broke and reversed in slope so that the magnitudes of the lift and pitching moment began to decrease with increasing sideslip. At zero sideslip, the swept-back wings possessed, in general, the largest magnitudes of BC_L/α and BC_m/α and the trapezoidal and rectangular plan forms the smallest. With Mach number fixed, C_L/α and C_m/α increased with increasing aspect ratio for most of the plan forms. The trapezoidal plan forms with raked-out tips proved to be exceptions to this rule in some cases. With the aspect ratio fixed, an increase in Mach number resulted in a decrease in BC_L/α and BC_m/α for most of the plan forms. The exceptions in this case were the swept-back plan forms.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., May 3, 1949.

APPENDIX A

FORMULAS FOR LIFT AND PITCHING MOMENT

DUE TO SIDESLIP

General Restriction: $\tan \beta \leq B$

TRIANGULAR WINGS

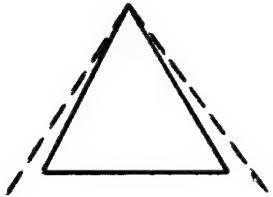
Subsonic Tips

$$0 < B_m < 1$$

$$C_L = \frac{L}{qS} = \frac{L}{qm c_r^2}$$

$$C_M = \frac{M}{qSc} = \frac{M}{\frac{2}{3} q m c_r^3}$$

$$m \geq \frac{1}{B + \sqrt{B^2+1}} \quad (\text{See footnote 1.})$$

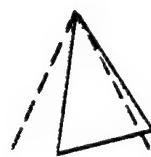
Phase 1, $0 \leq \tan \beta \leq \left(\frac{1-B_m}{B+m} \right)$ 

$$L = L_A$$

$$M = - L_A \frac{2}{3} c_r$$

Phase 2, $\left(\frac{1-B_m}{B+m} \right) \leq \tan \beta \leq m$

$$L = L_C$$



$$M = - L_C \frac{2}{3} c_r$$

¹Right tip crosses Mach cone before left tip crosses x axis.

Phase 3, $m \leq \tan \beta \leq \left(\frac{1+Bm}{B-m} \right)$

$$L = L_D$$

$$M = - L_D \frac{2}{3} c_r$$



$$m < \frac{1}{B + \sqrt{B^2 + 1}}$$



Phase 1, $0 \leq \tan \beta \leq m$

$$L = L_A$$

$$M = - L_A \frac{2}{3} c_r$$



Phase 2, $m \leq \tan \beta \leq \left(\frac{1-Bm}{B+m} \right)$

$$L = L_B$$

$$M = - L_B \frac{2}{3} c_r$$



Phase 3, $\left(\frac{1-Bm}{B+m} \right) \leq \tan \beta \leq \left(\frac{1+Bm}{B-m} \right)$

$$L = L_D$$

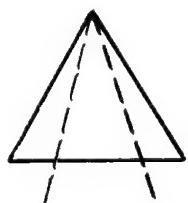
$$M = - L_D \frac{2}{3} c_r$$



Supersonic Tips

$$Bm \geq 1$$

$$C_L = \frac{L}{qS} = \frac{L}{qm c_r^2}$$



$$C_m = \frac{M}{qS c} = \frac{M}{\frac{2}{3} q m c_r^3}$$

Phase 1, $0 \leq \tan \beta \leq \left(\frac{B_m - 1}{B + m} \right)$

$$L = L_E$$

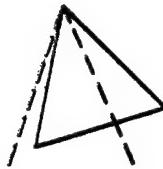
$$M = - L_E \frac{2}{3} c_r$$



Phase 2, $\left(\frac{B_m - 1}{B + m} \right) \leq \tan \beta \leq m$

$$L = L_C$$

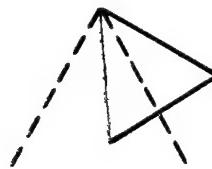
$$M = - L_C \frac{2}{3} c_r$$



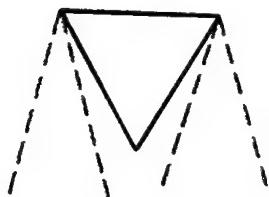
Phase 3, $m \leq \tan \beta \leq \left(\frac{1 + B_m}{B - m} \right)$

$$L = L_D$$

$$M = - L_D \frac{2}{3} c_r$$



$B_m \leq -1$

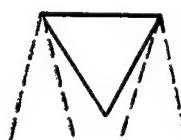


$$C_L = \frac{L}{qS} = - \frac{L}{qmc_r^2}$$

$$C_M = \frac{M}{qSc} = - \frac{M}{\frac{2}{3}qmc_r^3}$$

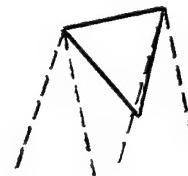
Phase 1, $0 \leq \tan \beta \leq - \left(\frac{B_m + 1}{B - m} \right)$

$$L = - \frac{4\alpha q c_r^2 m}{\sqrt{B^2 - \tan^2 \beta}}$$



$$M = - L \frac{c_r}{3} = + \frac{4\alpha q c_r^3 m}{3 \sqrt{B^2 - \tan^2 \beta}}$$

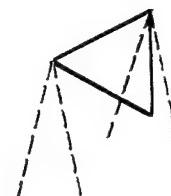
$$\text{Phase 2, } -\left(\frac{Bm+1}{B-m}\right) \leq \tan \beta \leq -m$$



$$L = L_F = L_{F_1} + L_{F_2}$$

$$M = -\left(L_{F_1} \bar{\xi}_{F_1} + L_{F_2} \bar{\xi}_{F_2}\right)$$

$$\text{Phase 3, } -m \leq \tan \beta \leq \left(\frac{1-Bm}{B+m}\right)$$



$$L = L_G = L_{G_1} + L_{G_2}$$

$$M = -\left(L_{G_1} \bar{\xi}_{G_1} + L_{G_2} \bar{\xi}_{G_2}\right)$$

SWEPT-BACK WINGS

Subsonic Tips

$$0 < Bm < 1$$

$$C_L = \frac{L}{qS} = \frac{L}{qmlc_r}$$

$$C_m = \frac{M}{qSc} = \frac{M}{\frac{2qmlc_r^2}{3}}$$

$$m \geq \frac{1}{B + \sqrt{B^2 + 1}}$$

$$0 \leq (l - c_r) \leq \frac{l(B^2 + 2Bm - 1)}{(-B^2 + 2\frac{B}{m} + 1)} \quad (\text{See footnote 2.})$$

²Right tip hits Mach cone from apex before trailing edge hits Mach cone from cutout.

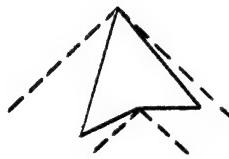


Phase 1, $0 \leq \tan \beta \leq \left(\frac{1-Bm}{B+m} \right)$

$$L = L_A - L_H$$

$$M = - \left(L_A \frac{2l}{3} - L_H \bar{\xi}_H \right)$$

$$\frac{l(B^2+2Bm-1)}{(-B^2+2\frac{B}{m}+1)} \leq (l-c_r) \leq Bml \quad (\text{See footnote 3.})$$



Phase 1, $0 \leq \tan \beta \leq \left[\frac{Blm - (l-c_r)}{B(l-c_r) + lm} \right]$

$$L = L_A - L_H$$

$$M = - \left(L_A \frac{2l}{3} - L_H \bar{\xi}_H \right)$$



$$m < \frac{1}{B + \sqrt{B^2 + 1}}$$

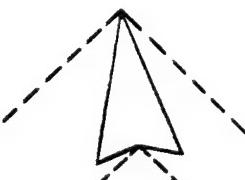
$$0 \leq (l-c_r) \leq \frac{l(B^2+2Bm-1)}{(-B^2+2\frac{B}{m}+1)} \quad (\text{See footnote 4.})$$



Phase 1, $0 \leq \tan \beta \leq m$

$$L = L_A - L_H$$

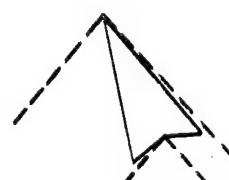
$$M = - \left(L_A \frac{2l}{3} - L_H \bar{\xi}_H \right)$$



Phase 2, $m \leq \tan \beta \leq \left(\frac{1-Bm}{B+m} \right)$

$$L = L_B - L_I$$

$$M = - \left(L_B \frac{2l}{3} - L_I \bar{\xi}_I \right)$$

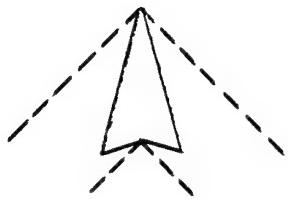


³ Mach cone at cutout does not cross wing trailing edge at zero side-slip.

⁴ Right tip hits Mach cone from apex before trailing edge hits Mach cone from cutout.

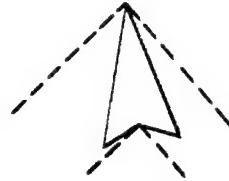
$$\frac{l(B^2+2Bm-1)}{(-B^2+2\frac{B}{m}+1)} \leq (l-c_r) \leq l_m \left(\frac{B-m}{Bm+1} \right) \quad (\text{See footnote 5.})$$

Phase 1, $0 \leq \tan \beta \leq m$



$$L = L_A - L_H$$

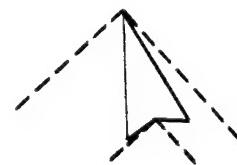
$$M = - \left(L_A \frac{2l}{3} - L_H \bar{\xi}_H \right)$$



$$\text{Phase 2, } m \leq \tan \beta \leq \left[\frac{Bl_m - (l-c_r)}{B(l-c_r) + l_m} \right]$$

$$L = L_B - L_I$$

$$M = - \left(L_B \frac{2l}{3} - L_I \bar{\xi}_I \right)$$



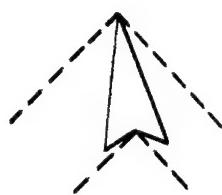
$$l_m \left(\frac{B-m}{Bm+1} \right) \leq (l-c_r) \leq Bl_m$$



$$\text{Phase 1, } 0 \leq \tan \beta \leq \left[\frac{Bl_m - (l-c_r)}{B(l-c_r) + l_m} \right]$$

$$L = L_A - L_H$$

$$M = - \left(L_A \frac{2l}{3} - L_H \bar{\xi}_H \right)$$



Supersonic Tips

$$Bm \geq 1$$

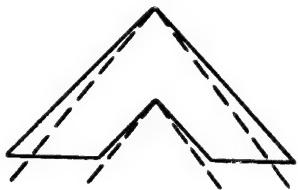
$$C_L = \frac{L}{qS} = \frac{L}{qm c_r (2l - c_r)}$$

$$C_M = \frac{M}{qS c} = \frac{M}{qm c_r^2 (2l - \frac{4}{3} c_r)}$$

⁵Left tip swings past x axis before trailing edge hits Mach cone from cutout.

$$m < \frac{2B}{B^2-1} \quad (\text{See footnote 6.})$$

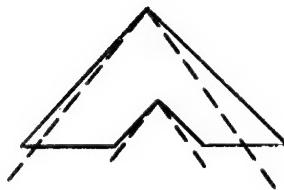
$$0 \leq (l - c_r) \leq \frac{l(-B^2 + \frac{2B}{m} + 1)}{(B^2 + 2Bm - 1)} \quad (\text{See footnote 7.})$$



Phase 1, $0 \leq \tan \beta \leq \left(\frac{Bm-1}{B+m} \right)$

$$L = L_E - L_J$$

$$M = - \left(L_E \frac{2}{3} l - M_J \right)$$



$$\frac{l(-B^2 + \frac{2B}{m} + 1)}{(B^2 + 2Bm - 1)} \leq (l - c_r) \leq \frac{l}{mB} \quad (\text{See footnote 8.})$$



Phase 1, $0 \leq \tan \beta \leq \left[\frac{l-Bm(l-c_r)}{Bl+m(l-c_r)} \right]$

$$L = L_E - L_J$$

$$M = - \left(L_E \frac{2}{3} l - M_J \right)$$



Phase 2, $\left[\frac{l-Bm(l-c_r)}{Bl+m(l-c_r)} \right] \leq \tan \beta \leq \left(\frac{Bm-1}{B+m} \right)$

$$L = L_E - L_K$$

$$M = - \left(L_E \frac{2}{3} l - M_K \right)$$



⁶ Left tip hits Mach cone before ξ axis crosses Mach cone at right.

⁷ Left tip hits apex Mach cone before right edge of cutout hits apex Mach cone.

⁸ Cutout does not overlap apex Mach cone at zero sideslip.

TRAPEZOIDAL WINGS

Subsonic Tips

$$-1 < B_m < 0$$

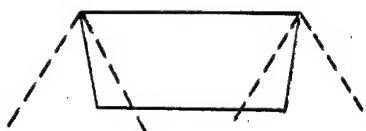
Span limitation

$$\tan \beta \leq \frac{B(b+c_r m) - c_r}{Bc_r + b + c_r m}$$

$$C_L = \frac{L}{qS} = \frac{L}{qc_r(b+mc_r)}$$

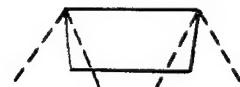
$$C_M = \frac{M}{qSc} = \frac{M}{qc_r^2(b+\frac{4}{3}mc_r)}$$

$$-m \leq \frac{1}{B + \sqrt{B^2 + 1}}$$

Phase 1, $0 \leq \tan \beta \leq -m$

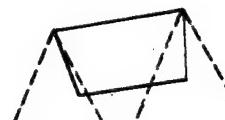
$$L = L_0 + L_S + L_N$$

$$M = - \left(L_0 \frac{2}{3} c_r + L_S \bar{\xi}_S + L_N \frac{2}{3} c_r \right)$$

Phase 2, $-m \leq \tan \beta \leq \left(\frac{1+B_m}{B-m} \right)$

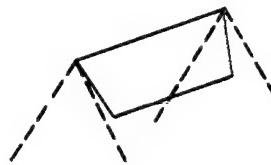
$$L = L_P + L_S + L_N$$

$$M = - \left(L_P \frac{2}{3} c_r + L_S \bar{\xi}_S + L_N \frac{2}{3} c_r \right)$$

Phase 3, $\left(\frac{1+B_m}{B-m} \right) \leq \tan \beta \leq \left(\frac{1-B_m}{B+m} \right)$

$$L = L_P + L_T$$

$$M = - \left(L_P \frac{2}{3} c_r + L_T \bar{\xi}_T \right)$$

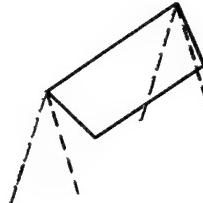


Phase 4, $\left(\frac{1-Bm}{B+m}\right) \leq \tan \beta \leq \frac{\text{Span}}{\text{limitation}}$

$$L = L_V + L_Q + L_T$$

$$M = -\left(L_V \frac{2c_r}{3} + L_Q \frac{2c_r}{3} + L_T \bar{\xi}_T\right)$$

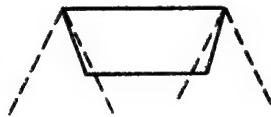
$$-m > \frac{1}{B + \sqrt{B^2 + 1}}$$



Phase 1, $0 \leq \tan \beta \leq \left(\frac{1+Bm}{B-m}\right)$

$$L = L_O + L_S + L_N$$

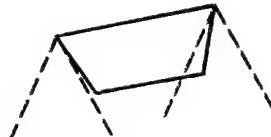
$$M = -\left(L_O \frac{2c_r}{3} + L_S \bar{\xi}_S + L_N \frac{2c_r}{3}\right)$$



Phase 2, $\left(\frac{1+Bm}{B-m}\right) \leq \tan \beta \leq -m$

$$L = L_O + L_T$$

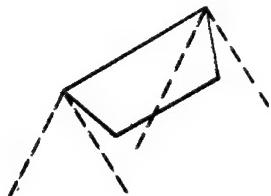
$$M = -\left(L_O \frac{2c_r}{3} + L_T \bar{\xi}_T\right)$$



Phase 3, $-m \leq \tan \beta \leq \left(\frac{1-Bm}{B+m}\right)$

$$L = L_P + L_T$$

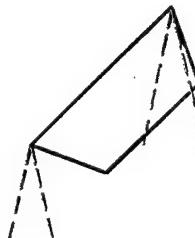
$$M = -\left(L_P \frac{2c_r}{3} + L_T \bar{\xi}_T\right)$$



Phase 4, $\left(\frac{1-Bm}{B+m}\right) \leq \tan \beta \leq \frac{\text{Span}}{\text{limitation}}$

$$L = L_V + L_Q + L_T$$

$$M = -\left(L_V \frac{2c_r}{3} + L_Q \frac{2c_r}{3} + L_T \bar{\xi}_T\right)$$



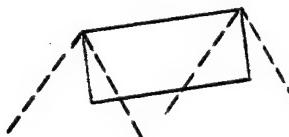
$B_m=0$ (Rectangular)

Span limitation

$$\tan \beta \leq \frac{Bb - c_r}{Bc_r + b}$$

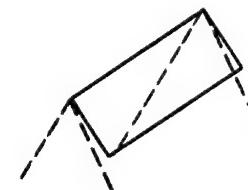
$$C_L = \frac{L}{qS} = \frac{L}{qb c_r}$$

$$C_m = \frac{M}{qS\bar{c}} = \frac{M}{qc_r^2 b}$$

Phase 1, $0 \leq \tan \beta \leq \frac{1}{B}$ 

$$L = L_P + L_S + L_N$$

$$M = - \left(L_P \frac{2}{3} c_r + L_S \bar{\epsilon}_S + L_N \frac{2}{3} c_r \right)$$

Phase 2, $\frac{1}{B} \leq \tan \beta \leq$ Span limitation

$$L = L_V + L_Q + L_T$$

$$M = - \left(L_V \frac{2}{3} c_r + L_Q \frac{2}{3} c_r + L_T \bar{\epsilon}_T \right)$$

 $0 < B_m < 1$

Span limitation

$$\tan \beta \leq \frac{B(b - mc_r) - c_r}{Bc_r + b - mc_r}$$

$$C_L = \frac{L}{qS} = \frac{L}{qc_r(b - mc_r)}$$

$$C_m = \frac{M}{qS\bar{c}} = \frac{M}{qc_r^2(b - \frac{4}{3}mc_r)}$$

$$m \leq \frac{1}{B + \sqrt{B^2 + 1}}$$

Phase 1, $0 \leq \tan \beta \leq m$ 

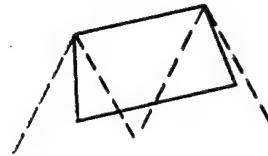
$$L = L_P + L_R + L_M$$

$$M = - \left(L_P \frac{2}{3} c_r + L_R \bar{\epsilon}_R + L_M \frac{2}{3} c_r \right)$$

Phase 2, $m \leq \tan \beta \leq \left(\frac{1-Bm}{B+m} \right)$

$$L = L_P + L_R + L_N$$

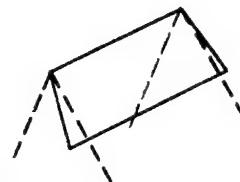
$$M = - \left(L_P \frac{2}{3} c_r + L_R \bar{\xi}_R + L_N \frac{2}{3} c_r \right)$$



Phase 3, $\left(\frac{1-Bm}{B+m} \right) \leq \tan \beta \leq \left(\frac{1+Bm}{B-m} \right)$

$$L = L_V + L_Q + L_R + L_N$$

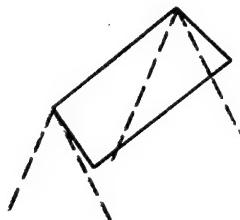
$$M = - \left(L_V \frac{2}{3} c_r + L_Q \frac{2}{3} c_r + L_R \bar{\xi}_R + L_N \frac{2}{3} c_r \right)$$



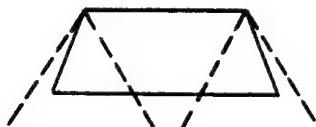
Phase 4, $\left(\frac{1+Bm}{B-m} \right) \leq \tan \beta \leq$ Span limitation

$$L = L_V + L_Q + L_T$$

$$M = - \left(L_V \frac{2}{3} c_r + L_Q \frac{2}{3} c_r + L_T \bar{\xi}_T \right)$$



$$m > \frac{1}{B + \sqrt{B^2 + 1}}$$



Phase 1, $0 \leq \tan \beta \leq \left(\frac{1-Bm}{B+m} \right)$

$$L = L_P + L_R + L_M$$

$$M = - \left(L_P \frac{2}{3} c_r + L_R \bar{\xi}_R + L_M \frac{2}{3} c_r \right)$$



Phase 2, $\left(\frac{1-Bm}{B+m} \right) \leq \tan \beta \leq m$

$$L = L_V + L_Q + L_R + L_M$$

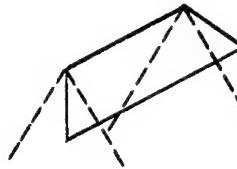
$$M = - \left(L_V \frac{2}{3} c_r + L_Q \frac{2}{3} c_r + L_R \bar{\xi}_R + L_M \frac{2}{3} c_r \right)$$



$$\text{Phase 3, } m \leq \tan \beta \leq \left(\frac{1+Bm}{B-m} \right)$$

$$L = L_V + L_Q + L_R + L_N$$

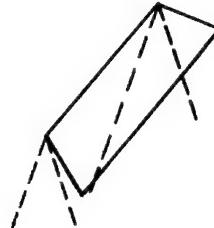
$$M = - \left(L_V \frac{2}{3} c_r + L_Q \frac{2}{3} c_r + L_R \bar{\epsilon}_R + L_N \frac{2}{3} c_r \right)$$



$$\text{Phase 4, } \left(\frac{1+Bm}{B-m} \right) \leq \tan \beta \leq \text{Span limitation}$$

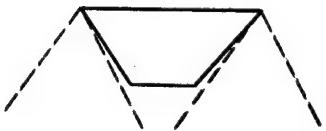
$$L = L_V + L_Q + L_T$$

$$M = - \left(L_V \frac{2}{3} c_r + L_Q \frac{2}{3} c_r + L_T \bar{\epsilon}_T \right)$$



Supersonic Tips

$$Bm \leq -1$$



Span limitation

$$\tan \beta \leq \frac{B(m c_r + b) - c_r}{B c_r + b + m c_r}$$

$$C_L = \frac{L}{qS} = \frac{L}{q c_r (b + m c_r)}$$

$$C_M = \frac{M}{qS^c} = \frac{M}{q c_r^2 (b + \frac{4}{3} m c_r)}$$

$$\text{Phase 1, } 0 \leq \tan \beta \leq - \left(\frac{Bm+1}{B-m} \right)$$

$$L = \frac{4 q \alpha c_r (c_r m + b)}{\sqrt{B^2 - \tan^2 \beta}}$$

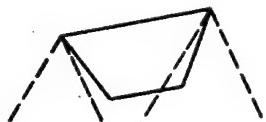


$$M = - L \left[\frac{c_r (4 c_r m + 3 b)}{6 (c_r m + b)} \right]$$

$$\text{Phase 2, } - \left(\frac{Bm+1}{B-m} \right) \leq \tan \beta \leq -m$$

$$L = L_0 + L_T$$

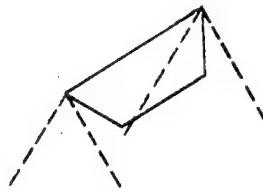
$$M = - \left(L_0 \frac{2}{3} c_r + L_T \bar{\epsilon}_T \right)$$



Phase 3, $-m \leq \tan \beta \leq \left(\frac{1-Bm}{B+m} \right)$

$$L = L_P + L_T$$

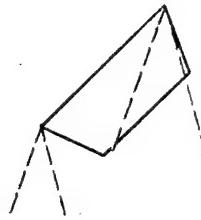
$$M = - \left(L_P \frac{2}{3} c_r + L_T \bar{\xi}_T \right)$$



Phase 4, $\left(\frac{1-Bm}{B+m} \right) \leq \tan \beta \leq \text{Span limitation}$

$$L = L_V + L_Q + L_T$$

$$M = - \left(L_V \frac{2}{3} c_r + L_Q \frac{2}{3} c_r + L_T \bar{\xi}_T \right)$$



$$Bm \geq 1$$

Span limitation

$$\tan \beta \leq \frac{B(b-mc_r)-c_r}{Bc_r+b-mc_r}$$



$$C_L = \frac{L}{qS} = \frac{L}{qc_r(b-mc_r)}$$

$$C_m = \frac{M}{qSc} = \frac{M}{qc_r^2(b - \frac{4}{3}mc_r)}$$

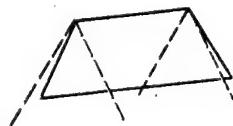
Phase 1, $0 \leq \tan \beta \leq \left(\frac{Bm-1}{B+m} \right)$



$$L = L_V + L_Q + L_R + L_L + L_U$$

$$M = - \left(L_V \frac{2}{3} c_r + L_Q \frac{2}{3} c_r + L_R \bar{\xi}_R + L_L \frac{2}{3} c_r + L_U \frac{2}{3} c_r \right)$$

Phase 2, $\left(\frac{Bm-1}{B+m} \right) \leq \tan \beta \leq m$



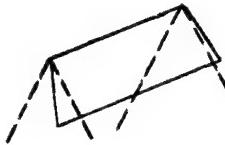
$$L = L_V + L_Q + L_R + L_M$$

$$M = - \left(L_V \frac{2}{3} c_r + L_Q \frac{2}{3} c_r + L_R \bar{\xi}_R + L_M \frac{2}{3} c_r \right)$$

$$\text{Phase 3, } m \leq \tan \beta \leq \left(\frac{1+Bm}{B-m} \right)$$

$$L = L_V + L_Q + L_R + L_N$$

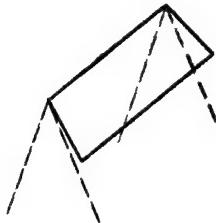
$$M = - \left(L_V \frac{2}{3} c_r + L_Q \frac{2}{3} c_r + L_R \bar{\xi}_R + L_N \frac{2}{3} c_r \right)$$



$$\text{Phase 4, } \left(\frac{1+Bm}{B-m} \right) \leq \tan \beta \leq \frac{\text{Span}}{\text{limitation}}$$

$$L = L_V + L_Q + L_T$$

$$M = - \left(L_V \frac{2}{3} c_r + L_Q \frac{2}{3} c_r + L_T \bar{\xi}_T \right)$$



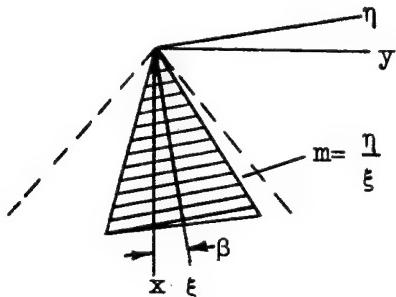
APPENDIX B

SUMMARY OF EXPRESSIONS FOR LIFT AND ESSENTIAL PITCHING MOMENTS
OR CHORDWISE CENTER-OF-PRESSURE LOCATIONS

Expressions apply to shaded areas on wings.

TRIANGULAR WINGS

A.



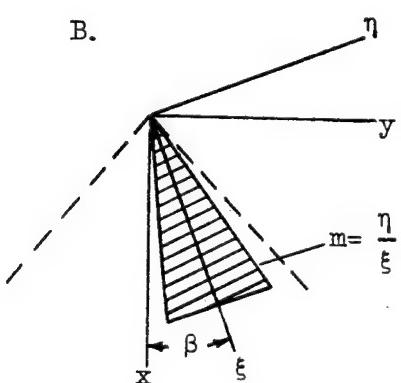
$$L_A = \frac{2\pi\alpha q c_r^2 m \cos \beta}{E} \sqrt{\frac{Gm}{B}}$$

where

E is the complete elliptic integral of the second kind with modulus $k = \sqrt{1-G^2}$

$$G = \frac{1}{2Bm(1+\tan^2\beta)} \left\{ (1-m^2\tan^2\beta) + B^2(m^2-\tan^2\beta) - \sqrt{[(1+m\tan\beta)^2-B^2(m-\tan\beta)^2][(1-m\tan\beta)^2-B^2(m+\tan\beta)^2]} \right\}$$

B.



$$L_B = \frac{2\pi\alpha q c_r^2 m P}{B} \sqrt{\frac{1-m\tan\beta}{1+m\tan\beta}}$$

when $\tan\beta = m$

$$P = \frac{1}{E} \sqrt{\frac{GBm}{1-m^2}}$$

where

E is the complete elliptic integral of the second kind defined under I_A .

When

$$\tan \beta = \frac{1-Bm}{B+m}$$

$$P = \frac{1}{\pi} \sqrt{\frac{2(1+m \tan \beta)}{(1+m \tan \beta) + B(m - \tan \beta)}}$$

when

$$m < \tan \beta < \frac{1-Bm}{B+m}$$

$$P = \sqrt{\frac{[B(m+\tan \beta)-G_1(l-m \tan \beta)](l+m \tan \beta)}{[G_1(l+m \tan \beta)+B(m-\tan \beta)](l-m \tan \beta)(1-G_1^2)}}$$

$$\left\{ \frac{\frac{G_1+k'}{k'K}}{\frac{G_1}{k'} + E + \frac{k' \sqrt{1-G_1^2}}{\sqrt{G_1^2-k'^2}} [E F(\varphi, k) - K E(\varphi, k)]} \right\}$$

$$G_1 = \frac{1}{2B(l+m^2)\tan \beta} \left\{ (1-m^2\tan^2 \beta) - B^2(m^2 - \tan^2 \beta) - \sqrt{[(l+m \tan \beta)^2 - B^2(m - \tan \beta)^2][(l - m \tan \beta)^2 - B^2(m + \tan \beta)^2]} \right\}$$

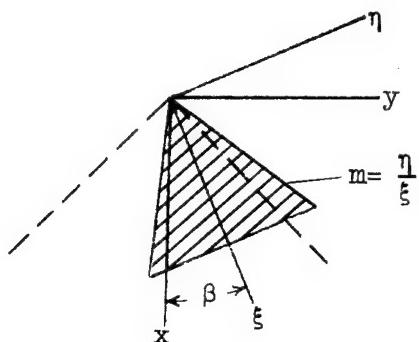
$$k = \sqrt{1-k'^2} \quad k' = \frac{G_1(l+m \tan \beta) + B(m - \tan \beta)}{(l+m \tan \beta) + G_1 B(m - \tan \beta)}$$

$$\varphi = \sin^{-1} \frac{\sqrt{G_1^2 - k'^2}}{G_1 k}$$

where

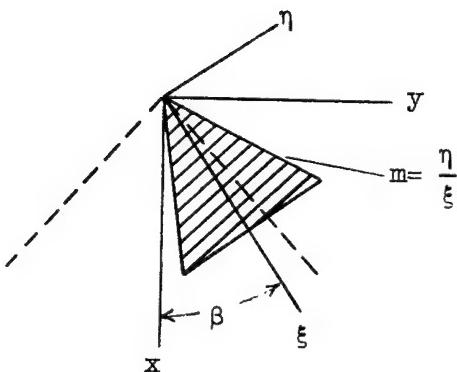
K and E are the complete elliptic integrals of the first and second kinds, respectively, with modulus k .

C.



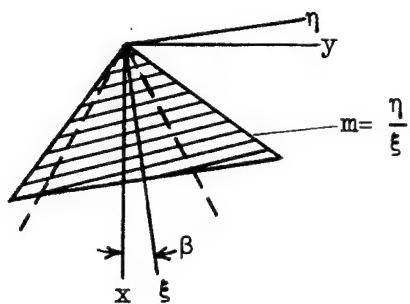
$$L_C = \frac{4\sqrt{2} \alpha q c_r^2 m^{3/2}}{\sqrt{(B + \tan \beta)[(1+B \tan \beta) + m(B - \tan \beta)]}}$$

D.



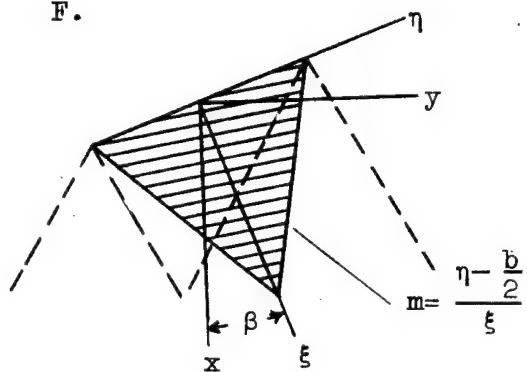
$$L_D = \frac{2\sqrt{2} \alpha q c_r^2 \sqrt{m} (m + \tan \beta)}{\sqrt{(B + \tan \beta)[(1+B \tan \beta) + m(B - \tan \beta)]}}$$

E.



$$L_E = \frac{4\alpha q c_r^2 m}{\sqrt{B^2 - \tan^2 \beta}}$$

F.



$$L_F = L_{F_1} + L_{F_2} = \frac{2\alpha q b^2}{\sqrt{[-2m(B+\tan \beta)][(1+B \tan \beta)-m(B-\tan \beta)]}}$$

where

$$L_{F_1} = \frac{2\alpha q b^2}{\sqrt{B+\tan \beta}} \left\{ \frac{1}{\sqrt{-2m[(1+B \tan \beta)-m(B-\tan \beta)]}} \right. \\ \left. \frac{\sqrt{B-\tan \beta}}{(1+B \tan \beta)-m(B-\tan \beta)} \right\}$$

and

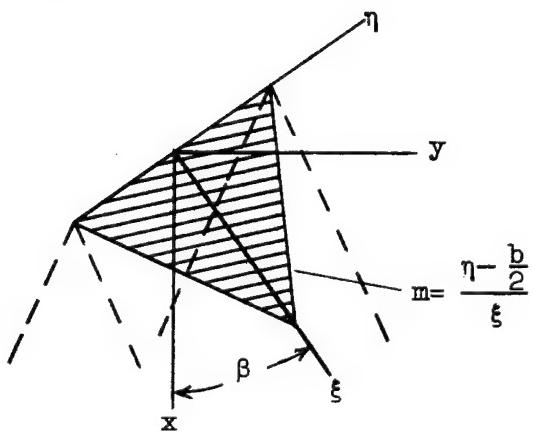
$$I_{F_2} = \frac{2\alpha q b^2}{\sqrt{B+\tan \beta}} \left[\frac{\sqrt{B-\tan \beta}}{(1+B \tan \beta)-m(B-\tan \beta)} \right]$$

$$\bar{\xi}_{F_1} = \frac{-b}{3 \left\{ (1+B \tan \beta)-m(B-\tan \beta)-\sqrt{-2m(B-\tan \beta)[(1+B \tan \beta)-m(B-\tan \beta)]} \right\}}$$

$$\left\{ \frac{(1+B \tan \beta)-3m(B-\tan \beta)}{4m} + \frac{(B-\tan \beta) \sqrt{-2m(B-\tan \beta)[(1+B \tan \beta)-m(B-\tan \beta)]}}{(1+B \tan \beta)-m(B-\tan \beta)} \right\}$$

$$\bar{\xi}_{F_2} = \frac{b(B-\tan \beta)}{3[(1+B \tan \beta)-m(B-\tan \beta)]}$$

G.



$$L_G = L_{G_1} + L_{G_2} = \frac{2aqb^2}{\sqrt{B+\tan \beta}} \left[\frac{1}{\sqrt{-2m[(1+B \tan \beta)-m(B-\tan \beta)]}} \right. \\ \left. \left\{ 1 - \frac{2(m+\tan \beta)}{(1+B \tan \beta)+m(B-\tan \beta)} \left[\sqrt{B-\tan \beta} + \frac{(1+B \tan \beta)-3m(B-\tan \beta)}{4m} \right] \right\} + \right. \\ \left. \frac{2\sqrt{B-\tan \beta}}{(1+B \tan \beta)-m(B-\tan \beta)} \right]$$

where

$$L_{G_1} = \frac{2aqb^2}{\sqrt{B+\tan \beta}} \left[\frac{1}{\sqrt{-2m[(1+B \tan \beta)-m(B-\tan \beta)]}} \right. \\ \left. \left\{ 1 - \frac{2(m+\tan \beta)}{(1+B \tan \beta)+m(B-\tan \beta)} \left[\sqrt{B-\tan \beta} + \frac{(1+B \tan \beta)-3m(B-\tan \beta)}{4m} \right] \right\} + \right. \\ \left. \frac{\sqrt{B-\tan \beta}}{(1+B \tan \beta)-m(B-\tan \beta)} \right]$$

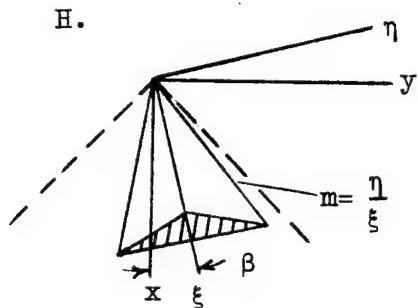
and

$$L_{G_2} = \frac{2aqb^2 \sqrt{B-\tan \beta}}{[(1+B \tan \beta)-m(B-\tan \beta)] \sqrt{B+\tan \beta}} \\ \bar{\xi}_{G_1} = \frac{b}{12m} \left[\frac{1}{2m(1+B \tan \beta)-(B-\tan \beta) \left\{ (\tan \beta) + m \left[1+2m-4m \sqrt{\frac{(1+B \tan \beta)-m(B-\tan \beta)}{-2m(B-\tan \beta)}} \right] \right\}} \right.$$

$$\left. \left\{ (1+B \tan \beta)(m+3\tan \beta)+m(B-\tan \beta) \left[-(5\tan \beta)+m-8m \sqrt{\frac{-2m(B-\tan \beta)}{(1+B \tan \beta)-m(B-\tan \beta)}} \right] \right\} \right]$$

$$\bar{\xi}_{G_2} = \frac{b(B-\tan \beta)}{3[(1+B \tan \beta)-m(B-\tan \beta)]}$$

SWEPT-BACK WING COMPONENTS

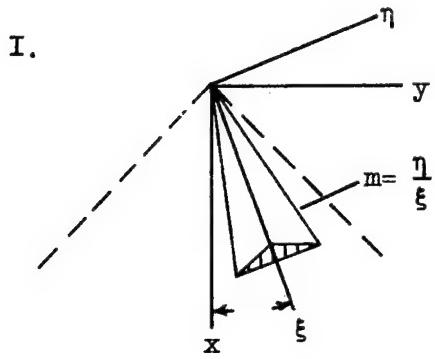


$$L_H = \frac{2\alpha q l^2 m \cos \beta}{E} \sqrt{\frac{Gm}{B}} \left\{ \pi - \frac{2c_r}{2l-c_r} \left[\frac{l-c_r}{l} + \right. \right.$$

$$\left. \frac{l}{\sqrt{c_r(2l-c_r)}} \left(\frac{\pi}{2} + \sin^{-1} \frac{l-c_r}{l} \right) \right] \left. \right\}$$

where G and E are defined under L_A .

$$\begin{aligned} \bar{\xi}_H = \frac{2l}{3} & \left\{ \pi - \frac{c_r^2}{(2l-c_r)^2} \left[-1 + \frac{c_r}{l} \left(\frac{c_r}{l} - 3 \right) + \frac{3l}{c_r} + \right. \right. \\ & \left. \left. \frac{1}{\sqrt{c_r(2l-c_r)}} \left(c_r - 2l + \frac{3l^2}{c_r} \right) \left(\frac{\pi}{2} + \sin^{-1} \frac{l-c_r}{l} \right) \right] \right\} \\ & \left\{ \frac{1}{\pi - \frac{2c_r}{2l-c_r} \left[\frac{l-c_r}{l} + \frac{l}{\sqrt{c_r(2l-c_r)}} \left(\frac{\pi}{2} + \sin^{-1} \frac{l-c_r}{l} \right) \right]} \right\} \end{aligned}$$

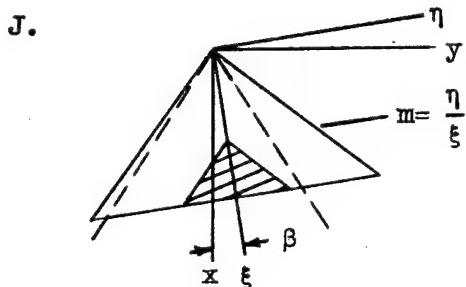


$$L_I = \frac{2\alpha q l^2 m P}{B} \sqrt{\frac{1-m \tan \beta}{1+m \tan \beta}}$$

$$\left\{ \pi - \frac{2c_r}{2l-c_r} \left[\frac{l-c_r}{l} + \frac{l}{\sqrt{c_r(2l-c_r)}} \right. \right. \\ \left. \left. \left(\frac{\pi}{2} + \sin^{-1} \frac{l-c_r}{l} \right) \right] \right\}$$

where P is a factor defined under L_B

$$\bar{\xi}_I = \bar{\xi}_H$$



$$\tan \beta < \left(\frac{Bm-l}{B+m} \right)$$

$$L_J = L_{J_1} + L_{J_2}$$

$$L_{J_1} = \frac{\alpha q m}{\pi} \left[\frac{1}{\sqrt{B^2(m+\tan \beta)^2 - (l-m \tan \beta)^2}} \right]$$

$$\left\{ -(m+\tan \beta)(2l-c_r)^2 \right.$$

$$\sin^{-1} \frac{l[(1-m \tan \beta) - B^2(m + \tan \beta) \tan \beta] + m(l - c_r)[(1-m \tan \beta)(\tan \beta) + B^2(m + \tan \beta)]}{Bm(1 + \tan^2 \beta)(2l - c_r)} -$$

$$c_{r^2} \left[(m-tan\beta) + (m+tan\beta) \right] - \left\{ 3 - \frac{2m[(1-m\tan\beta)(\tan\beta) + B^2(m+\tan\beta)]}{B^2(m+\tan\beta)^2 - (1-m\tan\beta)^2} \right\}$$

$$\sin^{-1} \frac{(1-m \tan \beta) - B^2(m + \tan \beta) \tan \beta}{Bm(1 + \tan^2 \beta)} + \frac{cr^2}{\sqrt{B^2(m - \tan \beta)^2 - (1 + m \tan \beta)^2}} \left\{ -1 \left[(m + \tan \beta) + \right. \right.$$

$$(\text{m}-\tan \beta) \left\{ 3 + \frac{2\text{m}[(1+\text{m}\tan \beta)(\tan \beta)-\text{B}^2(\text{m}-\tan \beta)]}{\text{B}^2(\text{m}-\tan \beta)^2-(1+\text{m}\tan \beta)^2} \right\} \sin^{-1} \frac{(1+\text{m}\tan \beta)+\text{B}^2(\text{m}-\tan \beta)\tan \beta}{\text{Bm}(1+\tan^2 \beta)} +$$

$$\left(\frac{m+tan\beta}{c_r} + \frac{2m[(1+m\tan\beta)(\tan\beta)-B^2(m-\tan\beta)]}{B^2(m-\tan\beta)^2-(1+m\tan\beta)^2} \right)$$

$$\sin^{-1} \frac{l[(1+m \tan \beta) + B^2(m - \tan \beta)] + m(1 - c_r)[(1+m \tan \beta)(\tan \beta) - B^2(m - \tan \beta)]}{Bm(1 + \tan^2 \beta)c_r}$$

$$- \frac{2c_r^2}{B^2(m+tan\beta)^2-(1-m\tan\beta)^2} + \frac{m\tan\beta}{B^2(m+tan\beta)^2-(1+m\tan\beta)^2} + \left[\frac{m\tan\beta}{B^2(m+tan\beta)^2-(1-m\tan\beta)^2} - \right] \sqrt{1-B^2\tan^2\beta}$$

$$\frac{2c_r(m+\tan \beta)}{B^2(m+\tan \beta)^2 - (1+m \tan \beta)^2} \sqrt{l^2(1-B^2\tan^2\beta) + 2ml(l-c_r)(B^2+1)(\tan \beta) - m^2(l-c_r)^2(B^2-\tan^2\beta)} -$$

$$\frac{4l^2}{\sqrt{B^2-\tan^2\beta}} \sin^{-1} \frac{l(B^2+1)(\tan \beta) - m(l-c_r)(B^2-\tan^2\beta)}{\sqrt{B}(1+\tan^2\beta)}$$

$$LJ_2 = \frac{\alpha q m}{\pi} \left[\frac{c_r^2}{\sqrt{B^2(m+\tan \beta)^2 - (1-m \tan \beta)^2}} \right] (m-\tan \beta) +$$

$$(m+\tan \beta) \left\{ \frac{4l}{c_r} - \frac{2m[(1-m \tan \beta)(\tan \beta) + B^2(m+\tan \beta)]}{B^2(m+\tan \beta)^2 - (1-m \tan \beta)^2} \right\}$$

$$\sin^{-1} \frac{l[(1-m \tan \beta) - B^2(m+\tan \beta)\tan \beta] - m(l-c_r)[(1-m \tan \beta)(\tan \beta) + B^2(m+\tan \beta)]}{Bm(1+\tan^2\beta)c_r} +$$

$$\left[\frac{m+\tan \beta}{\sqrt{B^2(m+\tan \beta)^2 - (1-m \tan \beta)^2}} + \frac{m-\tan \beta}{\sqrt{B^2(m+\tan \beta)^2 - (1+m \tan \beta)^2}} \right]$$

$$2\pi(l-c_r)^2 - \frac{(m-\tan \beta)(2l-c_r)^2}{\sqrt{B^2(m+\tan \beta)^2 - (1+m \tan \beta)^2}}$$

$$\frac{\sin^{-1} l [(1+m \tan \beta) + B^2(m-tan \beta) \tan \beta] - m(l-c_r) [(1+m \tan \beta)(\tan \beta) - B^2(m-tan \beta)]}{Bm(1+tan^2 \beta)(2l-c_r)} -$$

$$\frac{2c_r(m+tan \beta)}{B^2(m+tan \beta)^2 - (1-m \tan \beta)^2} \sqrt{l^2(1-B^2 \tan^2 \beta) - 2ml(l-c_r)(B^2+1)(\tan \beta) - m^2(l-c_r)^2(B^2 - \tan^2 \beta)} +$$

$$\frac{4l^2}{\sqrt{B^2 - tan^2 \beta}} \sin^{-1} \frac{l(B^2+1)(\tan \beta) + m(l-c_r)(B^2 - \tan^2 \beta)}{lB(1+tan^2 \beta)}$$

$$M_J = M_{J_1} + M_{J_2}$$

$$M_{J_1} = \frac{\alpha q m}{3\pi} \left\{ \frac{1}{\sqrt{B^2(m+tan \beta)^2 - (1-m \tan \beta)^2}} \left[-2(m+tan \beta) \left[4l^2 - c_r \left(3l^2 - \frac{c_r^2}{4} \right) \right] \right. \right.$$

$$\sin^{-1} l [(1-m \tan \beta) + B^2(m+tan \beta) \tan \beta] + m(l-c_r) [(1-m \tan \beta)(\tan \beta) + B^2(m+tan \beta)] -$$

$$\frac{Bm(1+tan^2 \beta)(2l-c_r)}{B^2(m+tan \beta)(m-tan \beta) - m(m+tan \beta)}$$

$$c_r^3 \left\{ (4m+3 \tan \beta) + \frac{m}{B^2(m+tan \beta)^2 - (1-m \tan \beta)^2} \left[(1-m \tan \beta) \tan \beta \right. \right. +$$

$$\left\{ \frac{3[(1-m)\tan \beta)(\tan \beta)+B^2(m+\tan \beta)]^2}{B^2(m+\tan \beta)^2-(1-m\tan \beta)^2} - \frac{(B^2-\tan^2\beta)}{(B^2-\tan^2\beta)} \right\} \sin^{-1} \frac{(1-m\tan \beta)-B^2(m+\tan \beta)\tan \beta}{Bm(1+\tan^2\beta)} -$$

$$\frac{8l^3}{\sqrt{B^2-\tan^2\beta}} \quad \frac{\sin^{-1} l(B^2+1)(\tan \beta)-m(l-c_r)(B^2-\tan^2\beta)}{lB(1+\tan^2\beta)} + \frac{1}{\sqrt{B^2(m-\tan \beta)^2-(1+m\tan \beta)^2}}$$

$$\left[-2c_r \left\{ \frac{c_r^2(m+\tan \beta)}{2} \left\{ -\frac{1}{2} + \frac{m[(1+m\tan \beta)(\tan \beta)-B^2(m-\tan \beta)]}{B^2(m-\tan \beta)^2-(1+m\tan \beta)^2} \right\} + \right. \right.$$

$$(m-\tan \beta) \left[-3l^2 + \frac{c_r^2 m^2}{2[B^2(m-\tan \beta)^2-(1+m\tan \beta)^2]} \left\{ -(B^2-\tan^2\beta) + \right. \right.$$

$$\left. \left. \frac{3[(1+m\tan \beta)(\tan \beta)-B^2(m-\tan \beta)]^2}{B^2(m-\tan \beta)^2-(1+m\tan \beta)^2} \right\} \right]$$

$$\sin^{-1} \frac{l[(1+m\tan \beta)+B^2(m-\tan \beta)\tan \beta]+m(l-c_r)[(1+m\tan \beta)(\tan \beta)-B^2(m-\tan \beta)]}{Bm(1+\tan^2\beta)c_r} -$$

$$-c_r^3 \left\{ (4m-3\tan \beta) - \frac{m}{B^2(m-\tan \beta)^2-(1+m\tan \beta)^2} \left[(m+\tan \beta) \right. \right.$$

$$\left\{ (1+m \tan \beta)(\tan \beta - B^2(m-\tan \beta)) + m(m-\tan \beta) \left\{ -(B^2 - m \tan \beta) + \frac{3[(1+m \tan \beta)(\tan \beta - B^2(m-\tan \beta))]^2}{B^2(m-\tan \beta)^2 - (1+m \tan \beta)^2} \right\} \right\}$$

$$\sin^{-1} \left[\frac{(1+m \tan \beta) + B^2(m-\tan \beta) \tan \beta}{Bm(1+\tan^2 \beta)} \right] + 2c_r^3 \left[\frac{1}{B^2(m+\tan \beta)^2 - (1-m \tan \beta)^2} - \right.$$

$$\left. \frac{1}{B^2(m-\tan \beta)^2 - (1+m \tan \beta)^2} \right] (\tan \beta) + \frac{3m}{2} \left\{ \frac{(m+\tan \beta)[(1-m \tan \beta)(\tan \beta) + B^2(m+\tan \beta)]}{[B^2(m+\tan \beta)^2 - (1-m \tan \beta)^2]^2} - \right.$$

$$\left. \frac{(m-\tan \beta)[(1+m \tan \beta)(\tan \beta) - B^2(m-\tan \beta)]}{[B^2(m-\tan \beta)^2 - (1+m \tan \beta)^2]^2} \right\} \sqrt{1-B^2 \tan^2 \beta} + \frac{c_r^2}{B^2(m-\tan \beta)^2 - (1+m \tan \beta)^2}$$

$$\left[(m+\tan \beta) + (m-\tan \beta) \left\{ \frac{3m[(1+m \tan \beta)(\tan \beta) - B^2(m-\tan \beta)]}{B^2(m-\tan \beta)^2 - (1+m \tan \beta)^2} - \frac{l}{c_r} \right\} \right]$$

$$\left. \sqrt{l^2(1-B^2 \tan^2 \beta) + 2ml(l-c_r)(B^2+1)(\tan \beta) - m^2(l-c_r)^2(B^2 - \tan^2 \beta)} \right\}$$

$$M_{J_2} = \frac{\alpha q m}{3\pi} \left\{ \frac{-2(m-\tan \beta)}{\sqrt{B^2(m-\tan \beta)^2 - (1+m \tan \beta)^2}} \left[+ 4l^3 - c_r \left(3l^2 - \frac{c_r^2}{4} \right) \right] \right.$$

$$\sin^{-1} \frac{l[(1+m \tan \beta) + B^2(m-\tan \beta) \tan \beta] - m(l-c_r)[(1+m \tan \beta)(\tan \beta) + B^2(m-\tan \beta)]}{Bm(1+\tan^2 \beta)(2l-c_r)} +$$

$$\frac{8l^3}{\sqrt{B^2 - \tan^2 \beta}} \sin^{-1} \frac{l(B^2+1)(\tan \beta) + m(l-c_r)(B^2 - \tan^2 \beta)}{lB(1+\tan^2 \beta)} + \frac{c_r}{\sqrt{B^2(m+\tan \beta)^2 - (1-m \tan \beta)^2}}$$

$$\left\{ \frac{(m+\tan \beta) \left[6l^2 - \frac{c_r^2 m^2}{B^2(m+\tan \beta)^2 - (1-m \tan \beta)^2} \right]}{B^2(m+\tan \beta)^2 - (1-m \tan \beta)^2} - \frac{\left\{ 3[(1-m \tan \beta)(\tan \beta) + B^2(m+\tan \beta)] \right\}^2}{B^2(m+\tan \beta)^2 - (1-m \tan \beta)^2} - \right.$$

$$\left. \left(B^2 - \tan^2 \beta \right) \right\} + \frac{c_r^2(m-\tan \beta)}{2} \left\{ 1 + \frac{2m[(1-m \tan \beta)(\tan \beta) + B^2(m+\tan \beta)]}{B^2(m+\tan \beta)^2 - (1-m \tan \beta)^2} \right\}$$

$$\sin^{-1} \frac{l[(1-m \tan \beta) - B^2(m+\tan \beta) \tan \beta] - m(l-c_r)[(1-m \tan \beta)(\tan \beta) + B^2(m+\tan \beta)]}{Bm(1+\tan^2 \beta)c_r} +$$

$$\frac{c_r^2}{B^2(m+tan\beta)^2-(1-m\tan\beta)^2} \left[(m-tan\beta)-(m+tan\beta) \left\{ \frac{l}{c_r} + \frac{3m[(1-m\tan\beta)(tan\beta)+B^2(m+tan\beta)]}{B^2(m+tan\beta)^2-(1-m\tan\beta)^2} \right\} \right]$$

$$\sqrt{l^2(1-B^2\tan^2\beta)-2ml(l-c_r)(B^2+1)(tan\beta)-m^2(l-c_r)^2(B^2-tan^2\beta)}$$

$$+ \left[\frac{m-tan\beta}{\sqrt{B^2(m-tan\beta)^2-(1+m\tan\beta)^2}} + \frac{m+tan\beta}{\sqrt{B^2(m+tan\beta)^2-(1-m\tan\beta)^2}} \right] 2\pi [2l^3 - c_r(3l^2 - c_r^2)] \left\{ \right.$$

$$\tan\beta = \left(\frac{Bm-1}{B+m} \right) \quad (\text{See footnote 1.})$$

$$L_{J_1} = \frac{\alpha q}{\pi} \left\{ \sqrt{\frac{m}{(Bm-1)(B+m)(B^2+1)}} \left[-\frac{(m^2+2Bm-1)(2l-c_r)^2}{2} \right] \right\}$$

$$\sin^{-1} \left[1 - \frac{2c_r(Bm-1)(B+m)}{(2l-c_r)B(1+m^2)} \right] - \frac{4l^2m(B+m)}{\sqrt{(B^2+1)(B^2+2Bm-1)}}$$

¹Left tip hits Mach cone from apex.

$$\begin{aligned}
& \sin^{-1} \left[\frac{c_r}{l} - 1 + \frac{c_r(Bm-1)(B+m)}{lB(1+m^2)} \right] + c_r^2 \sqrt{\frac{m}{(Bm-1)(B+m)(B^2+1)}} \\
& \left\{ -\frac{(1+m^2)}{2} + (m^2+2Bm-1) \left[\frac{B(1+m^2)}{4(Bm-1)(B+m)} - 1 \right] \right\} \sin^{-1} \left[1 - \frac{2(Bm-1)(B+m)}{B(1+m^2)} \right] + \\
& c_r^2 \left\{ (m^2+2Bm-1) \left[\frac{1}{2(Bm-1)(B+m)} + \frac{1}{B(1+m^2)} \right] - \frac{2}{3B} \left[\frac{(Bm-1)(B+m)}{B(1+m^2)} - \frac{5}{2} \right] \right\} \sqrt{\frac{2Bm-m^2(B^2-1)}{B^2+1}} + \\
& \frac{1}{B} \left[-\frac{2}{3} (5l-c_r) + \frac{c_r}{1+m^2} \left\{ (B^2+1)^{+(B+m)} \left[\frac{2(Bm-1)}{3B} - (B+m) \right] \right\} \right] \\
& \sqrt{\frac{c_r m (1+m^2) - c_r m (B^2+2Bm-1)}{B^2+1}} \Bigg\} \\
& L_{J_2} = \frac{\alpha q}{\pi} \left[c_r \sqrt{\frac{m}{(Bm-1)(B+m)(B^2+1)}} \left\{ 2l(m^2+2Bm-1) + \frac{c_r}{2} \left[-2(Bm-1) - \frac{B(1+m^2)(m^2+2Bm-1)}{2(Bm-1)(B+m)} \right] \right\} \right]
\end{aligned}$$

$$\sin^{-1} \left[1 - \frac{2(2l-c_r)(Bm-1)(B+m)}{c_r B(1+m^2)} \right] + \frac{4ml^2(B+m)}{\sqrt{(B^2+1)(B^2+2Bm-1)}}$$

$$\sin^{-1} \left[1 - \frac{c_r}{l} + \frac{(2l-c_r)(Bm-1)(B+m)}{lB(1+m^2)} \right] + \left\{ \frac{2l}{B} - \frac{c_r}{2} \left[\frac{m^2+2Bm-1}{(Bm-1)(B+m)} + \frac{2}{B} \right] \right\}$$

$$\sqrt{\frac{m(2l-c_r)[mc_r(B^2+2Bm-1)-2l(Bm-1)(B+m)]}{B^2+1}} + \sqrt{\frac{\pi}{(Bm-1)(B+m)(B^2+1)}} [\pi(l-c_r)^2(m^2+2Bm-1)]$$

$$M_{J_1} = \frac{\alpha q}{3\pi} \left\{ \sqrt{\frac{\pi}{(Bm-1)(B+m)(B^2+1)}} \left[c_r^3 \left\{ -(2m^2+3Bm-1) + \frac{B(1+m^2)}{4(Bm-1)(B+m)} \right\} \right. \right.$$

$$\left. \left. \left[\frac{m^2+4Bm-3}{2} + \frac{3B(1+m^2)(m^2+2Bm-1)}{8(Bm-1)(B+m)} \right] \sin^{-1} \left[1 - \frac{2(Bm-1)(B+m)}{B(1+m^2)} \right] - (m^2+2Bm-1) \right\} \right]$$

$$\left[4l^3 - c_r \left(3l^2 - \frac{c_r^2}{4} \right) \right] \sin^{-1} \left[1 - \frac{2c_r(Bm-1)(B+m)}{(2l-c_r)B(1+m^2)} \right] - \frac{8l^3m(B+m)}{\sqrt{(B^2+1)(B^2+2Bm-1)}}$$

$$\sin^{-1} \left[-1 + \frac{c_r}{l} + \frac{c_r(Bm-1)(B+m)}{lB(1+m^2)} \right] + 4 c_r^3 \left\{ \frac{77}{120B} + \frac{1}{5B(1+m^2)} \left[\frac{11m^2+46Bm-11}{24} + \frac{m}{B} \right. \right.$$

$$\left[\frac{1}{8(Bm-1)(B+m)} \left[\frac{3m^2+10Bm-7}{4} + \frac{3B(1+m^2)(m^2+2Bm-1)}{8(Bm-1)(B+m)} \right] \right]$$

$$\sqrt{\frac{2Bm-m^2(B^2-1)}{B^2+1}} + \frac{4}{2B} \left\{ -7l^2 + \frac{c_r l}{3} - \frac{c_r l}{3(1+m^2)} \left(\frac{m^2+6Bm-1}{4} + \frac{m}{B} \right) + \frac{c_r^2}{3} - \frac{c_r^2}{3(1+m^2)} \right\}$$

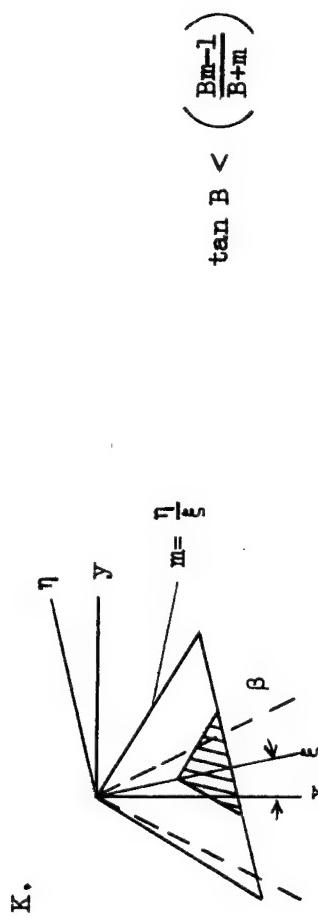
$$\left[\frac{9m^2+34Bm-9}{8} + \frac{2m}{B} + \frac{(Bm-1)(3+m)}{B(1+m^2)} \right] \left[\frac{\frac{m^2+6Bm-1}{4} + \frac{m}{B}}{\frac{c_{rm}[2Bl(1+m^2)-c_{rm}(B^2+2Bm-1)]}{B^2+1}} \right]$$

$$M_{J_2} = \alpha q \sqrt{\frac{\frac{m^2+2Bm-1}{m}}{(Bm-1)(B+m)(B^2+1)}} [2l^3 - c_r(3l^2 - c_r^2)] + \frac{1}{\pi} \sqrt{\frac{m}{(Bm-1)(B+m)(B^2+1)}}$$

$$\left\{ l^2(m^2+2Bm-1) + \frac{c_r^2(m^2-2Bm+3)}{12} - \frac{c_r^2B(1+m^2)}{12(Bm-1)(B+m)} \left[\frac{m^2+4Bm-3}{2} + \frac{3B(1+m^2)(m^2+2Bm-1)}{8(Bm-1)(B+m)} \right] \right\}$$

$$\sin^{-1} \left[1 - \frac{2(2l-c_r)(B_m-1)(B+m)}{c_r B(1+m^2)} \right] + \frac{8l^3 m(B+m)}{3\sqrt{(B^2+1)(B^2+2Bm-1)}}$$

$$\frac{c_r^2}{8(B_m-1)(B+m)} \left[\frac{m^2+6Bm-5}{4} + \frac{3B(1+m^2)(m^2+2Bm-1)}{8(B_m-1)(B+m)} \right] \left\{ \frac{\frac{m(2l-c_r)[c_r m(B^2+2Bm-1)-2l(B_m-1)(B+m)]}{B^2+1}}{B^2+1} \right\}$$



$$L_K = L_{J_1} + L_{K_1}$$

$$L_{K_1} = \alpha q m \left\{ \frac{2l^2}{\sqrt{B^2-\tan^2 \beta}} + \frac{1}{\sqrt{B^2(m+\tan \beta)^2-(1-m\tan \beta)^2}} + \frac{c_r^2(m-\tan \beta)}{2} + (m+\tan \beta) \right\}$$

$$\left\{ 2(l^2 - 3lc_r + c_r^2) + \frac{c_r^2 m [(1-m \tan \beta)(\tan \beta) + B^2(m + \tan \beta)]}{B^2(m + \tan \beta)^2 - (1-m \tan \beta)^2} \right\} +$$

$$\frac{c_r(m - \tan \beta)}{\sqrt{B^2(m - \tan \beta)^2 - (1+m \tan \beta)^2}} \left(\frac{3}{2} c_r - 2l \right) \Bigg\}$$

$$M_K = M_{J_1} + M_{K_1}$$

$$M_{K_1} = \frac{2\alpha q m}{3} \left[\frac{2l^3}{\sqrt{B^2 - \tan^2 \beta}} + \frac{m + \tan \beta}{\sqrt{B^2(m + \tan \beta)^2 - (1-m \tan \beta)^2}} \right] - \frac{9}{2} c_r l^2 + c_r^3$$

$$\left[1 - \frac{m^2}{4[B^2(m + \tan \beta)^2 - (1-m \tan \beta)^2]} \left\{ (B^2 - \tan^2 \beta) - \frac{3[(1-m \tan \beta)(\tan \beta) + B^2(m + \tan \beta)]^2}{B^2(m + \tan \beta)^2 - (1-m \tan \beta)^2} \right\} \right]^+$$

$$\frac{c_r(m-\tan \beta)}{2} \left[\frac{1}{\sqrt{B^2(m-\tan \beta)^2 - (1+m \tan \beta)^2}} \left(\frac{7}{4} c_r^2 - 3l^2 \right) - \frac{c_r^2}{2\sqrt{B^2(m+\tan \beta)^2 - (1-m \tan \beta)^2}} \right]$$

$$\left[\frac{m[(1-m \tan \beta)(\tan \beta) + B^2(m+\tan \beta)]}{B^2(m+\tan \beta)^2 - (1-m \tan \beta)^2} + \frac{1}{2} \right]$$

$$\tan \beta = \left(\frac{Bm-1}{B+m} \right) \quad (\text{see footnote 2})$$

$$I_{K_1} = \alpha q \left[\frac{2ml^2(B+m)}{\sqrt{(B^2+1)(B^2+2Bm-1)}} + \left\{ (m^2+2Bm-1) \left[l^2 - 3lc_r + \frac{5c_r^2}{4} + \frac{c_r^2 B(1+m^2)}{8(Bm-1)(B+m)} \right] - \frac{c_r^2(1+m^2)}{4} \right\} \right]$$

$$\sqrt{\frac{m}{(Bm-1)(B+m)(B^2+1)}}$$

²Left leading tip hits Mach cone from apex.

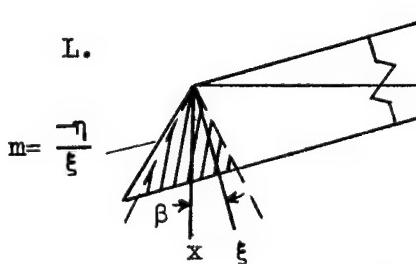
$$M_{K_1} = \alpha q \left\{ \frac{4ml^3(B+m)}{3\sqrt{(B^2+1)(B^2+2Bm-1)}} + \left[(m^2+2Bm-1) \left(\frac{2l^3}{3} - \frac{3l^2c_r}{2} \right) + \right. \right.$$

$$\left. \frac{c_r^3}{2} \left\{ \frac{31m^2+44Bm-19}{24} - \frac{m}{4B} + \frac{mB(1+m^2)}{4(Bm-1)} + \frac{1}{(Bm-1)(B+m)} \right. \right]$$

$$\left. \left. \left[-m(B^2+2Bm-1) \left(\frac{11m^2+8Bm-3}{24} - \frac{m}{4B} \right) + \frac{(m^2+2Bm-1)R^2(1+m^2)^2}{32(Bm-1)(B+m)} \right] \right] \right\}$$

$$\sqrt{\frac{m}{(Bm-1)(B+m)(B^2+1)}} \quad \left. \right\}$$

TRAPEZOIDAL WING COMPONENTS

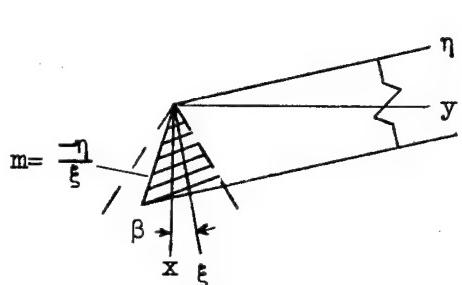


$$L = \frac{\eta}{\xi} \quad m = \frac{\eta}{\xi}$$

$$L_L = 2\alpha q c_r^2 \left\{ \frac{1}{\sqrt{B^2-\tan^2\beta}} \left[\frac{B(1+\tan^2\beta)}{B^2-\tan^2\beta} + (m-\tan\beta) \right] - \right.$$

$$\left. \frac{(m-\tan\beta)}{\sqrt{B^2(m-\tan\beta)^2(1+m\tan\beta)^2}} \left(m - \frac{1+B\tan\beta}{B-\tan\beta} \right) \right\}$$

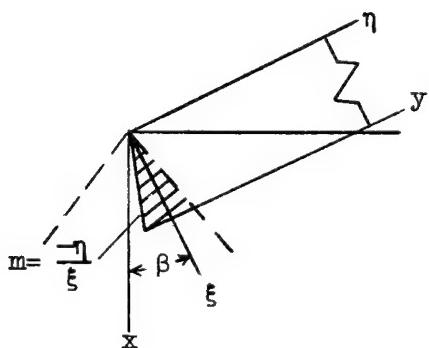
M.



$$I_M = \frac{\alpha q c_r^2}{\sqrt{B^2 - \tan^2 \beta}}$$

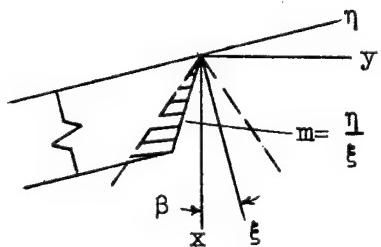
$$\left(3m + \frac{1 - 3B \tan \beta - 2 \tan^2 \beta}{B + \tan \beta} \right)$$

N.



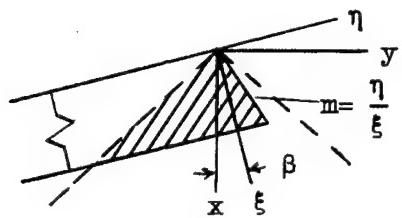
$$I_N = \frac{\alpha q c_r^2}{\sqrt{B^2 - \tan^2 \beta}} \left(m + \frac{1 - B \tan \beta}{B + \tan \beta} \right)$$

O.



$$I_O = \frac{\alpha q c_r^2}{\sqrt{B^2 - \tan^2 \beta}} \left(m + \frac{1 + B \tan \beta}{B - \tan \beta} \right)$$

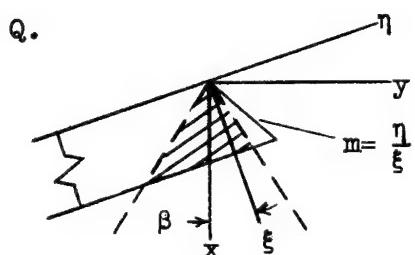
P.



$$L_p = \frac{\alpha q c_r^2}{\sqrt{B^2 - \tan^2 \beta}}$$

$$\left(3m + \frac{1+3B \tan \beta - 2 \tan^2 \beta}{B \tan \beta} \right)$$

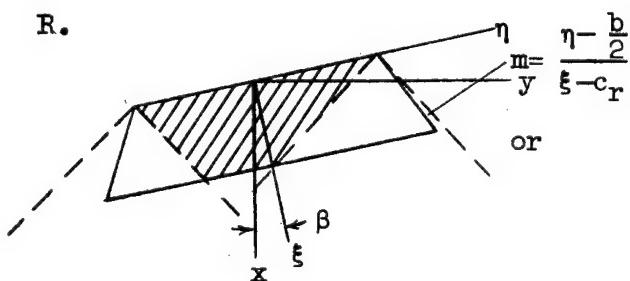
Q.



$$L_Q = 2\alpha q c_r^2 \left\{ \frac{1}{\sqrt{B^2 - \tan^2 \beta}} \left[\frac{B(1+\tan^2 \beta)}{B^2 - \tan^2 \beta} + (m + \tan \beta) \right] - \right.$$

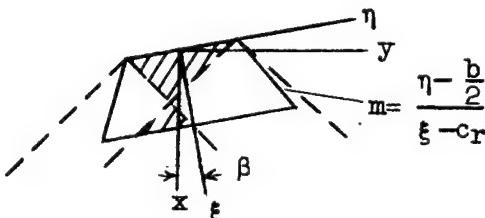
$$\left. \frac{(m + \tan \beta)}{\sqrt{B^2(m + \tan \beta)^2 - (1 - m \tan \beta)^2}} \left[m - \left(\frac{1 - B \tan \beta}{B + \tan \beta} \right) \right] \right\}$$

R.

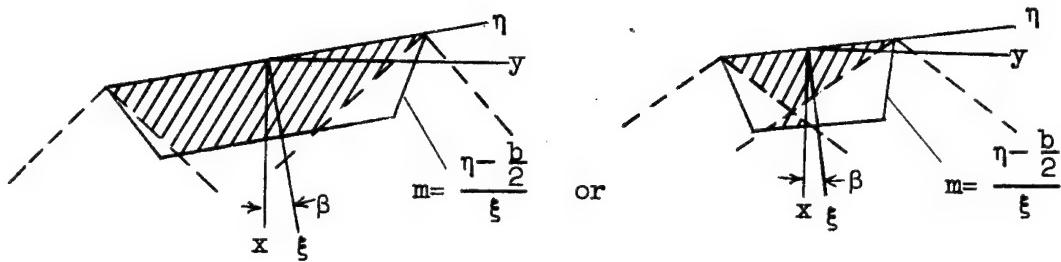


$$L_R = \frac{4\alpha q c_r}{\sqrt{B^2 - \tan^2 \beta}} \left[\frac{-c_r B (1 + \tan^2 \beta)}{B^2 - \tan^2 \beta} + (b - 2c_r m) \right]$$

$$\bar{\xi}_R = \frac{c_r}{6} \left[\frac{3(b - 2c_r m)(B^2 - \tan^2 \beta) - 4c_r B (1 + \tan^2 \beta)}{(b - 2c_r m)(B^2 - \tan^2 \beta) - c_r B (1 + \tan^2 \beta)} \right]$$



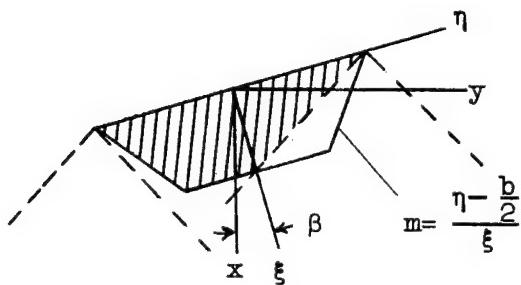
S.



$$L_S = \frac{4\alpha q c_r}{\sqrt{B^2 - \tan^2 \beta}} \left[\frac{-c_r B (1 + \tan^2 \beta)}{B^2 - \tan^2 \beta} + b \right]$$

$$\bar{\xi}_S = \frac{c_r}{6} \left[\frac{3b(B^2 - \tan^2 \beta) - 4c_r B(1 + \tan^2 \beta)}{b(B^2 - \tan^2 \beta) - c_r B(1 + \tan^2 \beta)} \right]$$

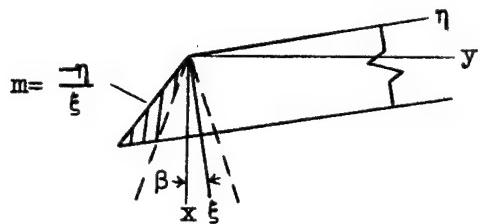
T.



$$L_T = \frac{2\alpha q c_r}{\sqrt{B^2 - \tan^2 \beta}} \left[2b + c_r m - c_r \left(\frac{1 + B \tan \beta}{B - \tan \beta} \right) \right]$$

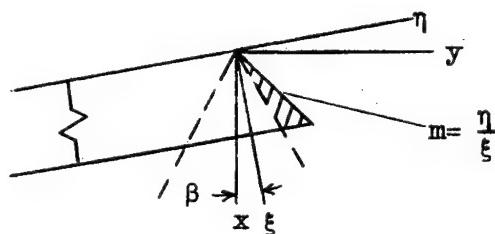
$$\bar{\xi}_T = \frac{c_r}{3} \left[\frac{(3b + 2c_r m)(B - \tan \beta) - 2c_r(1 + B \tan \beta)}{(2b + c_r m)(B - \tan \beta) - c_r(1 + B \tan \beta)} \right]$$

U.



$$L_U = \frac{2\alpha q c_r^2 (m - \tan \beta)}{\sqrt{B^2(m - \tan \beta)^2 - (1 + m \tan \beta)^2}} \left[m - \left(\frac{1 + B \tan \beta}{B - \tan \beta} \right) \right]$$

V.



$$L_V = \frac{2\alpha q c_r^2 (m + \tan \beta)}{\sqrt{B^2(m + \tan \beta)^2 - (1 - m \tan \beta)^2}} \left[m - \left(\frac{1 - B \tan \beta}{B + \tan \beta} \right) \right]$$

APPENDIX C

LIFT AND PITCHING MOMENT AS FUNCTIONS OF MACH NUMBER
AND PLAN-FORM PARAMETERS AT ZERO SIDESLIP

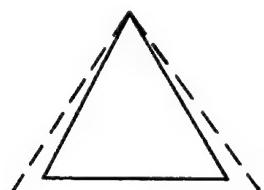
$$C_L = \frac{L}{qS}$$

$$C_m = \frac{M}{qSc}$$

TRIANGULAR WINGS

Subsonic Tips

$$0 < B_m < 1$$



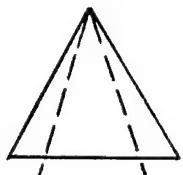
$$C_L = \frac{L}{q c_r^2 m} = \frac{\pi \alpha A}{2E}$$

$$C_m = \frac{M}{\frac{2}{3} q c_r^3 m} = \frac{-\pi \alpha A}{2E}$$

where E is the complete elliptic integral of the second kind with modulus $\sqrt{1-B_m^2} = \sqrt{1-\frac{B^2 A^2}{16}}$

Supersonic Tips

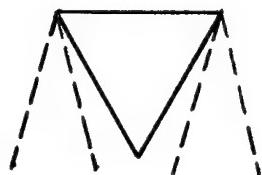
$$B_m \geq 1$$



$$C_L = \frac{L}{q c_r^2 m} = \frac{4\alpha}{B}$$

$$C_m = \frac{M}{\frac{2}{3} q c_r^3 m} = \frac{-4\alpha}{B}$$

$$B_m \leq -1$$



$$C_L = -\frac{L}{qc_r^2 m} = \frac{4\alpha}{B}$$

$$C_m = -\frac{M}{\frac{2}{3}qc_r^3 m} = \frac{-2\alpha}{B}$$

SWEPT-BACK WINGS

Subsonic Tips

$$0 < B_m < 1$$



$$C_L = \frac{L}{ql c_r m} = \frac{2m\alpha BA}{E(BA-2B_m)} \left\{ 1 - \frac{4B_m}{BA} + \frac{BA}{2\sqrt{2B_m(BA-2B_m)}} \left[\frac{\pi}{2} + \sin^{-1} \left(1 - \frac{4B_m}{BA} \right) \right] \right\}$$

$$C_m = \frac{M}{\frac{2}{3}ql c_r^2 m} = \frac{-m\alpha B^2 A^2}{2E(BA-2B_m)^2} \left\{ -1 - \frac{4B_m}{BA} \left(3 - \frac{4B_m}{BA} \right) + \frac{3BA}{4B_m} + \frac{BA}{2\sqrt{2B_m(BA-2B_m)}} \left(\frac{4B_m}{BA} - 2 + \frac{3BA}{4B_m} \right) \left[\frac{\pi}{2} + \sin^{-1} \left(1 - \frac{4B_m}{BA} \right) \right] \right\}$$

where E is the complete elliptic integral of the second kind with modulus $\sqrt{1-B^2 m^2}$

Supersonic Tips

$$B_m \geq 1$$



$$C_L = \frac{L}{q c_r m (2l - c_r)} = Aa \left[\frac{1}{B_m} - \frac{2}{\pi B_m} \sin^{-1} \frac{B_m \sqrt{BA(BA-4B_m)}}{BA} + \right.$$

$$\left. \frac{1}{\sqrt{B^2 m^2 - 1}} \left(\frac{4B_m}{BA} - 1 \right) + \frac{1}{\pi \sqrt{B^2 m^2 - 1}} \right]$$

$$\left\{ \frac{[BA + \sqrt{BA(BA-4B_m)}]^2}{2B^2 A^2} \sin^{-1} \frac{BA + B^2 m^2 \sqrt{BA(BA-4B_m)}}{B_m [BA + \sqrt{BA(BA-4B_m)}]} + \right.$$

$$\left. \frac{[BA - \sqrt{BA(BA-4B_m)}]}{BA} \left[\left\{ \frac{(B^2 m^2 + 1)[BA - \sqrt{BA(BA-4B_m)}]}{2(B^2 m^2 - 1)BA} - 2 \right\} \right] \right\}$$

$$\sin^{-1} \frac{BA - B^2 m^2 \sqrt{BA(BA-4B_m)}}{B_m [BA - \sqrt{BA(BA-4B_m)}]} + \frac{1}{\sqrt{B^2 m^2 - 1}} \left\{ -1 + \frac{\sqrt{BA(BA-4B_m)}}{BA} + \right.$$

$$\left. \frac{\sqrt{BA[B(A(1-B^2 m^2) + 4B^3 m^3)]}}{BA} + \frac{[BA - \sqrt{BA(BA-4B_m)}]}{BA} \left(2 - \frac{B^2 m^2}{B^2 m^2 - 1} \right) \right\}$$

$$\left. \sin^{-1} \frac{1}{B_m} \right] \left. \right\}$$

$$\begin{aligned}
C_m = \frac{M}{qc_r^2 m(2l - \frac{4}{3}c_r)} &= \frac{-qB^2 A^2}{\pi B [B^2 A^2 - 6BABm - \sqrt{BA(BA-4Bm)}] (BA-4Bm)} \\
&\quad \left\{ -2\pi + 4 \sin^{-1} \frac{Bm \sqrt{BA(BA-4Bm)}}{BA} + \frac{Bm}{\sqrt{B^2 m^2 - 1}} \right. \\
&\quad \left. \left[\frac{[BA - \sqrt{BA(BA-4Bm)}]^3}{2B^3 A^3} - \frac{\left\{ \frac{3B^2 m^2}{(B^2 m^2 - 1)^{3/2}} - \right.}{\left. \left. \right\} \right] \right. \\
&\quad \left[4 - \frac{B^2 m^2 (2+B^2 m^2)}{(B^2 m^2 - 1)^2} \right] \sin^{-1} \frac{1}{Bm} + \pi \left\{ \frac{BA-4Bm}{BA} \left[3 - \frac{\sqrt{BA(BA-4Bm)}}{BA} \right] \right\} + \left[-2 + \frac{3Bm}{BA} - \right. \\
&\quad \left. \left. \frac{\sqrt{BA(BA-4Bm)}}{BA} \left(2 + \frac{Bm}{BA} \right) \right] \sin^{-1} \frac{BA+B^2 m^2 \sqrt{BA(BA-4Bm)}}{Bm[BA+\sqrt{BA(BA-4Bm)}]} + \left[1 - \frac{\sqrt{BA(BA-4Bm)}}{BA} \right] \right. \\
&\quad \left. \left\{ 3 - \frac{[BA - \sqrt{BA(BA-4Bm)}]^2 [B^2 m^2 (B^2 m^2 + 6) - 1]}{4B^2 A^2 (B^2 m^2 - 1)^2} \right\} \sin^{-1} \frac{BA-B^2 m^2 \sqrt{BA(BA-4Bm)}}{Bm[BA-\sqrt{BA(BA-4Bm)}]} + \left\{ \left(1 - \frac{6B^2 m^2}{B^2 m^2 - 1} \right) \right. \right. \\
&\quad \left. \left. \left[1 - \frac{\sqrt{BA(BA-4Bm)}}{BA} \right] - \frac{4Bm}{BA} \left(1 - \frac{3B^2 m^2}{B^2 m^2 - 1} \right) \right\} \frac{\sqrt{BA[B(A-3B^2 m^2) + 4B^3 m^3]}}{2BA \sqrt{B^2 m^2 - 1}} \right\}
\end{aligned}$$

TRAPEZOIDAL WINGS

Subsonic Tips

$$-1 < B_m < 0$$



$$C_L = \frac{L}{qc_r(b+c_r m)} = \frac{4\alpha}{B} \left[\frac{B_m - 1 + BA + \sqrt{BA(BA+4B_m)}}{2B_m + BA + \sqrt{BA(BA+4B_m)}} \right]$$

$$C_m = \frac{M}{qc_r^2 \left(b + \frac{4}{3}c_r m \right)} = - \frac{2\alpha}{B} \left[\frac{\frac{4B_m - 4 + 3BA + 3\sqrt{BA(BA+4B_m)}}{8B_m + 3BA + 3}}{\sqrt{BA(BA+4B_m)}} \right]$$

$$B_m = 0 \quad (\text{Rectangular})$$



$$C_L = \frac{L}{qc_r b} = \frac{4\alpha}{B} \left(1 - \frac{1}{2BA} \right)$$

$$C_m = \frac{M}{qc_r^2 b} = - \frac{2\alpha}{B} \left(1 - \frac{2}{3BA} \right)$$

$$0 < B_m < 1$$



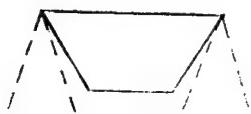
$$C_L = \frac{L}{qc_r(b-c_r m)} = \frac{4\alpha}{B} \left[\frac{B_m + 1 - BA - \sqrt{BA(BA-4B_m)}}{2B_m - BA - \sqrt{BA(BA-4B_m)}} \right]$$

$$C_m = \frac{M}{qc_r^2 \left(b - \frac{4}{3}c_r m \right)} = - \frac{2\alpha}{B} \left[\frac{\frac{4-3BA-3\sqrt{BA(BA-4B_m)}}{8B_m-3BA-3\sqrt{BA(BA-4B_m)}}}{\sqrt{BA(BA-4B_m)}} \right]$$

Supersonic Tips

$$B_m \leq -1$$

$$C_L = \frac{L}{qc_r(b+c_r m)} = \frac{4\alpha}{B}$$



$$C_m = \frac{M}{qc_r^2 \left(b + \frac{4}{3}c_r m \right)} = - \frac{2\alpha}{B}$$

$$B_m \geq 1$$



$$C_L = \frac{L}{qc_r(b - c_r m)} = \frac{4\alpha}{B}$$

$$C_m = \frac{M}{qc_r^2 \left(b - \frac{4}{3}c_r m \right)} = \frac{2\alpha}{B} \left[\frac{4Bm - 3BA - 3\sqrt{BA(BA - 4Bm)}}{8Bm - 3BA - 3\sqrt{BA(BA - 4Bm)}} \right]$$

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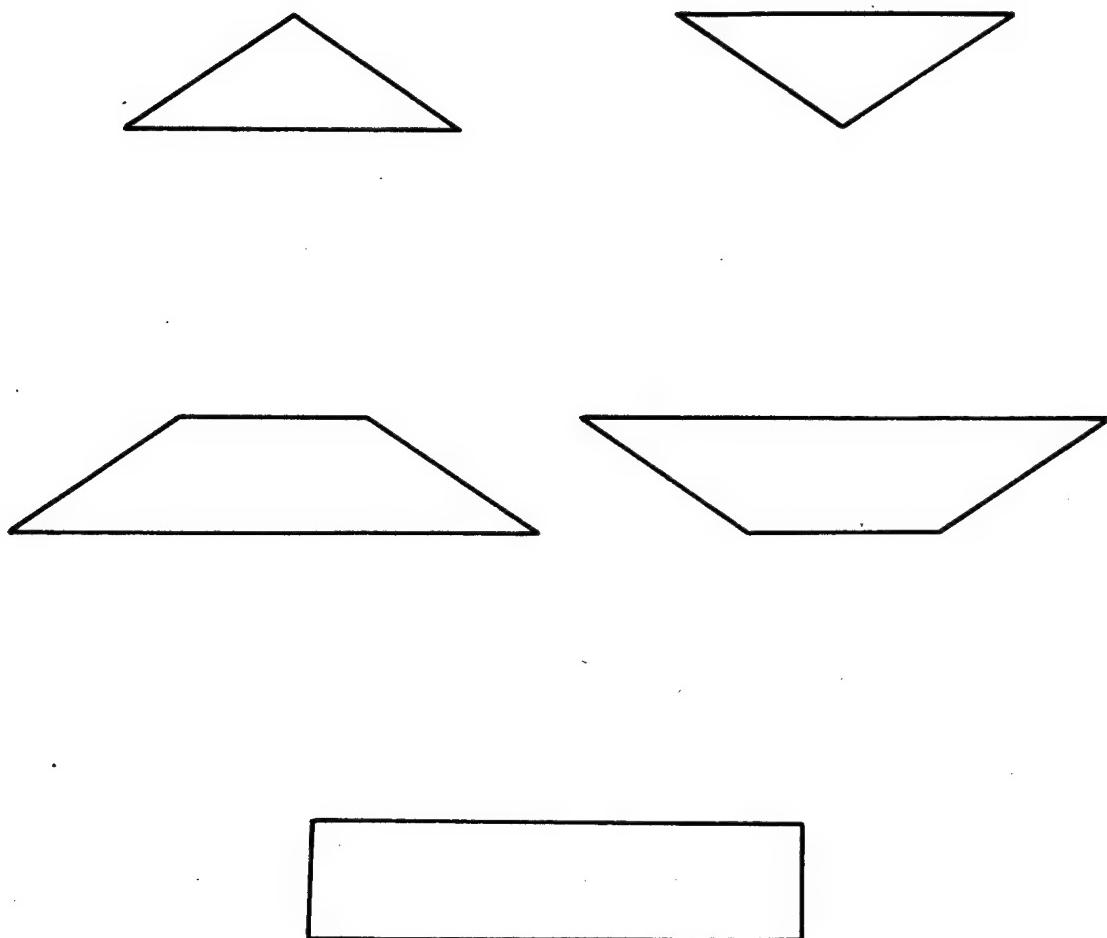


Figure 1.—The triangular, trapezoidal, and rectangular plan-form types investigated.

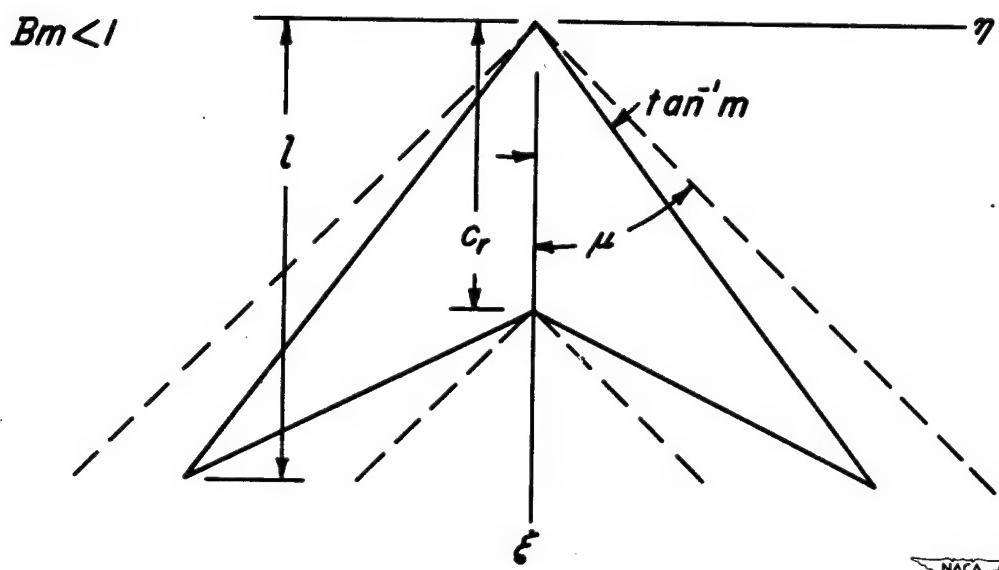
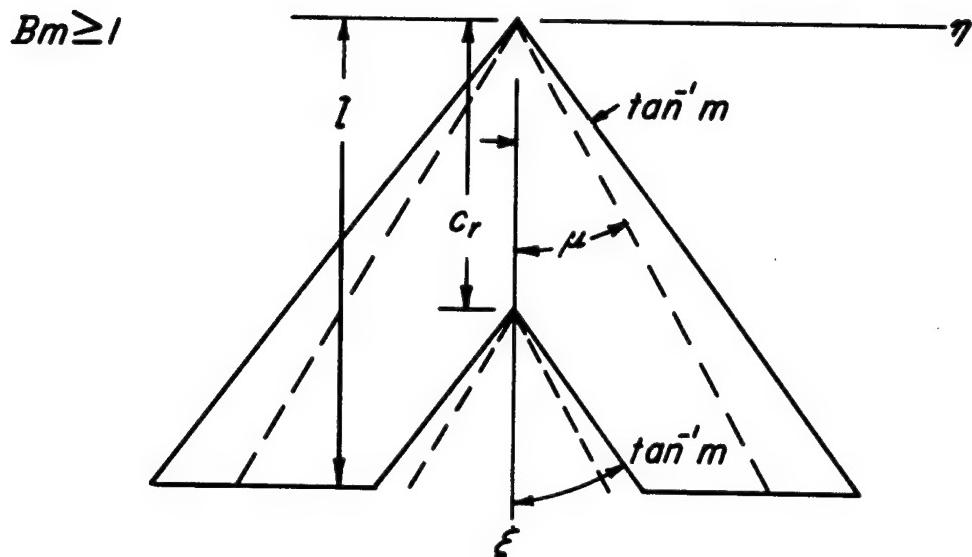


Figure 2.—Swept-back plan forms and Mach cone configurations investigated.

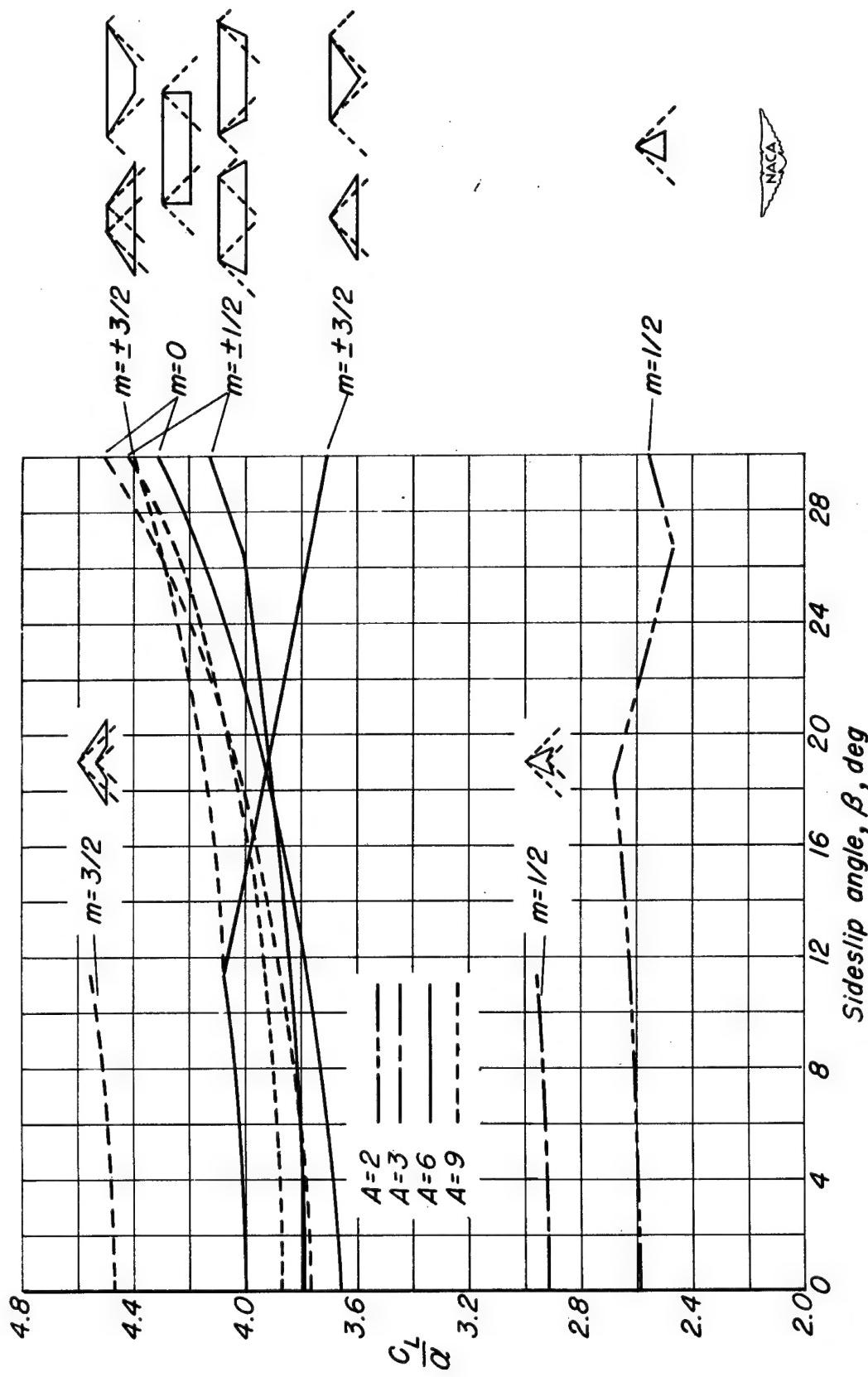


Figure 3.—Variation of lift coefficient per unit angle of attack with sideslip angle for $B=1$.

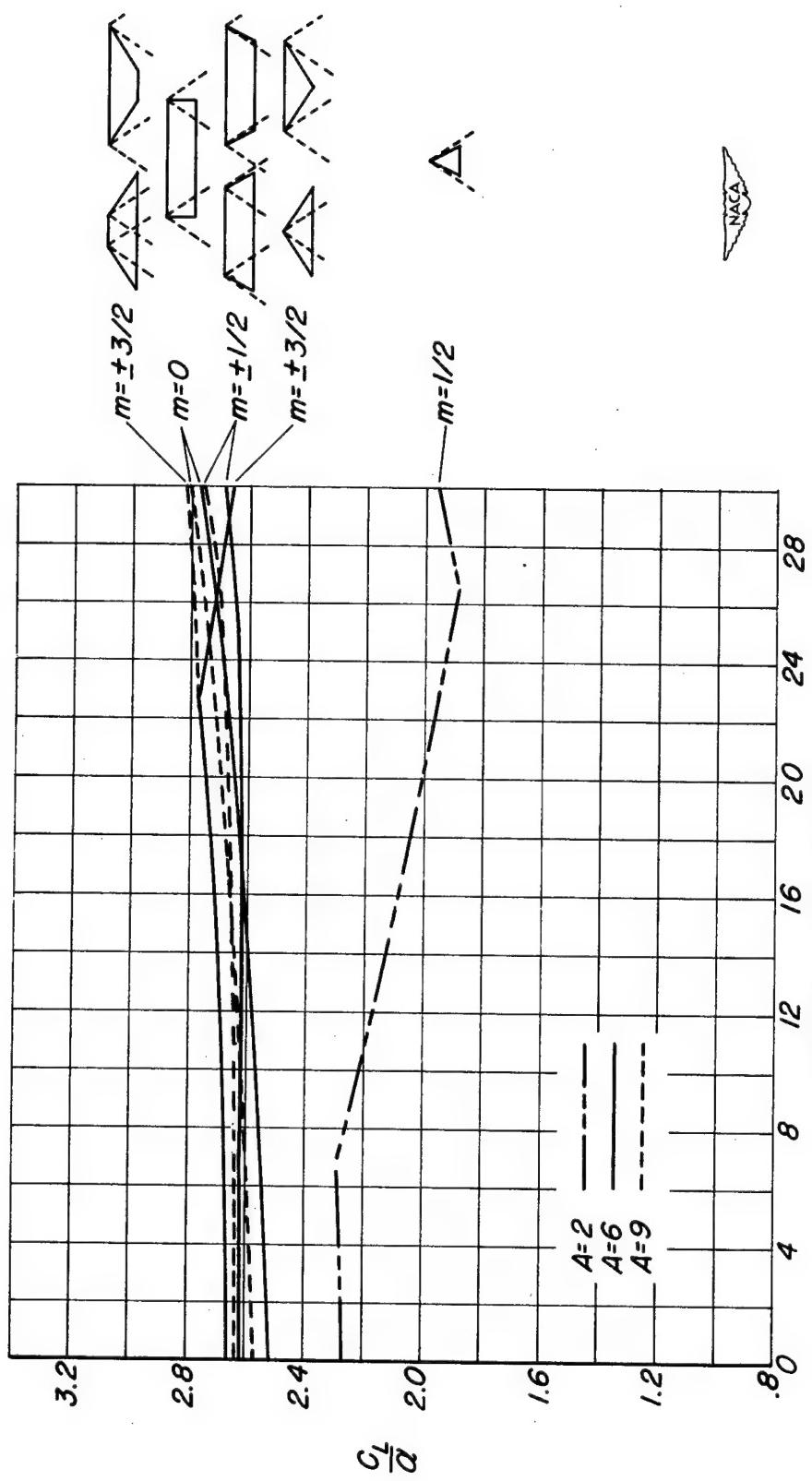
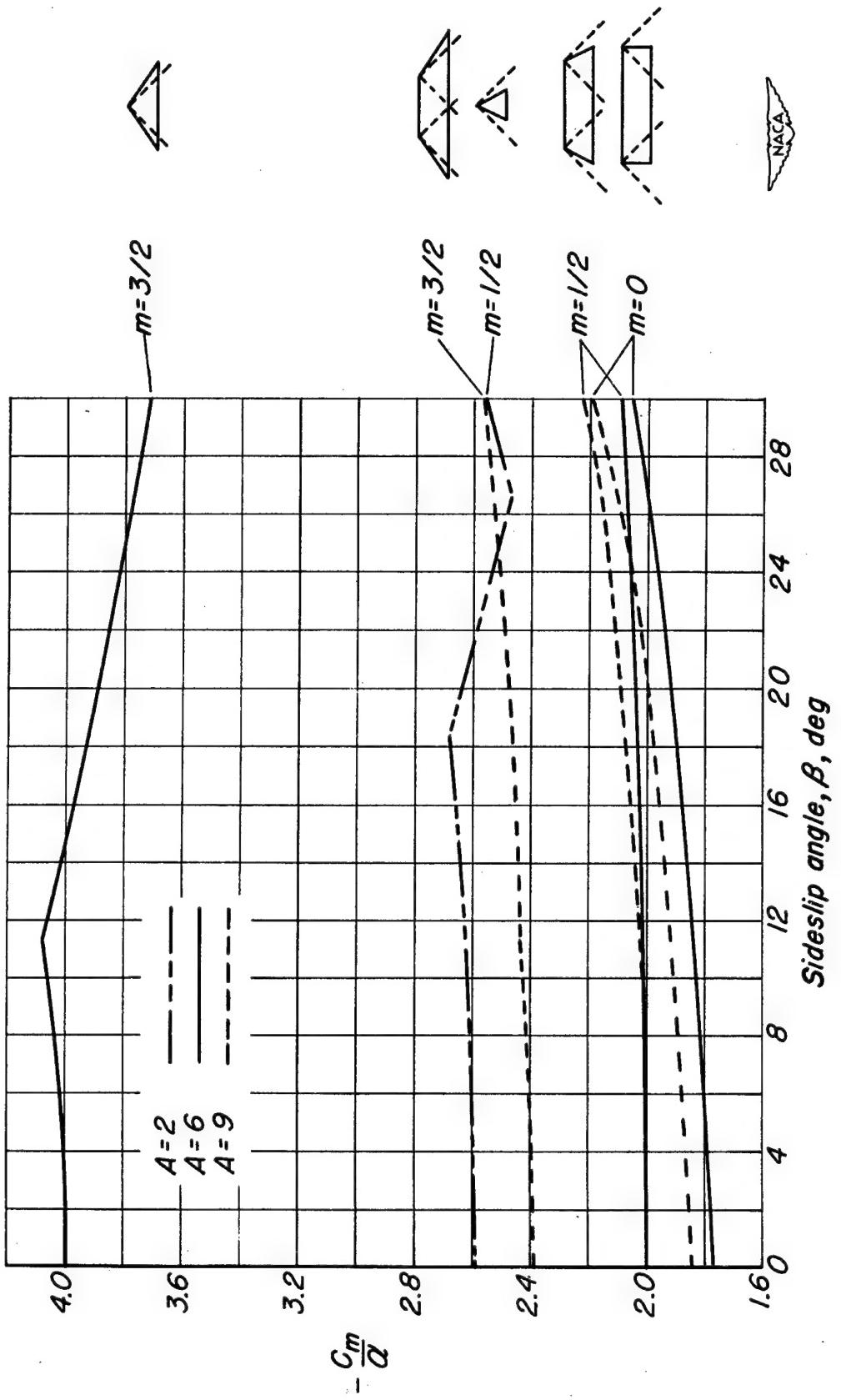
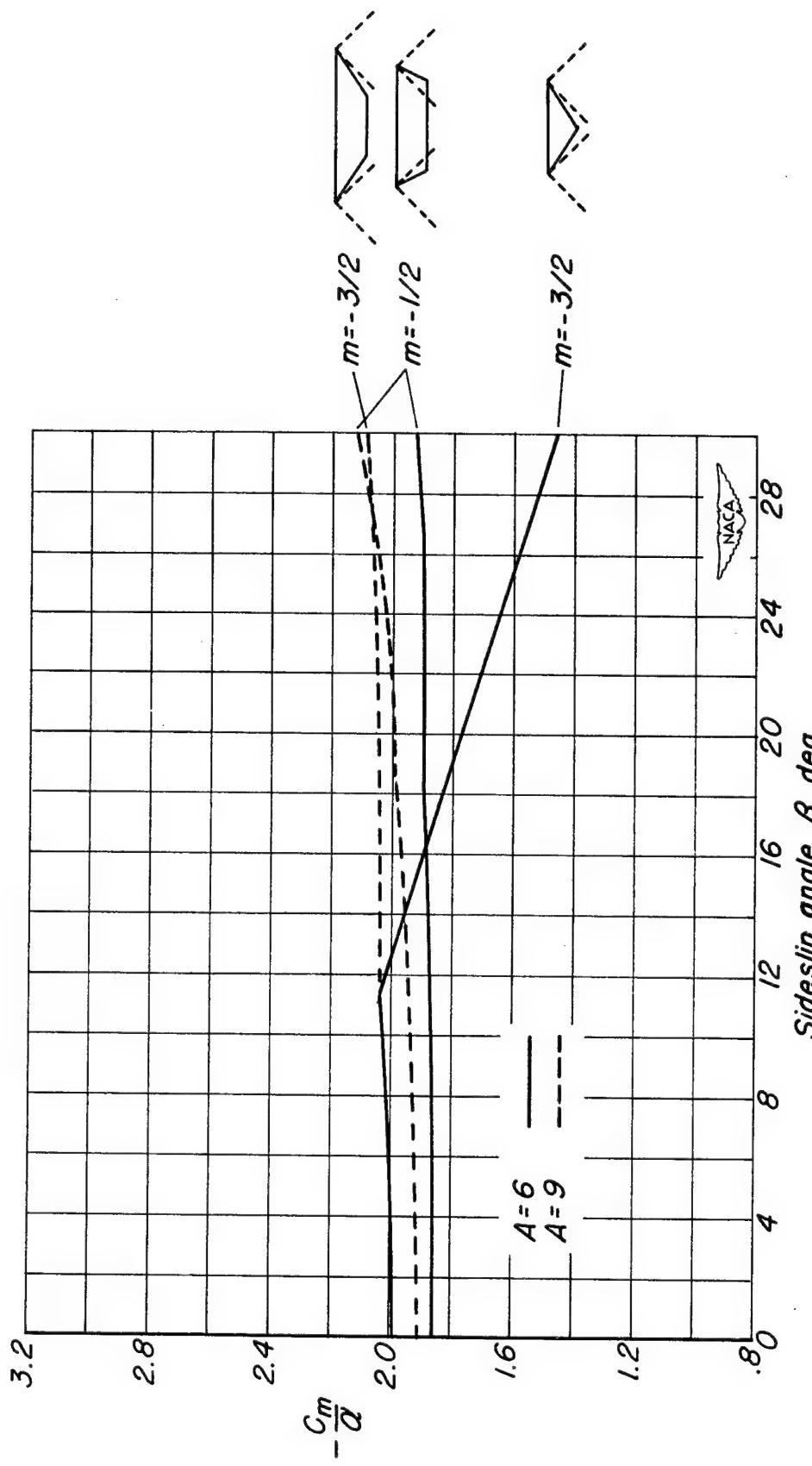


Figure 4.— Variation of lift coefficient per unit angle of attack with sideslip angle for $B=1.5$.

(a) Triangular, trapezoidal, and rectangular plan forms; $m \geq 0$.Figure 5.—Variation of pitching-moment coefficient per unit angle of attack with sideslip angle for $B=1$.



(b) Traingular and trapezoidal plan forms; $m < 0$.

Figure 5.- Continued.

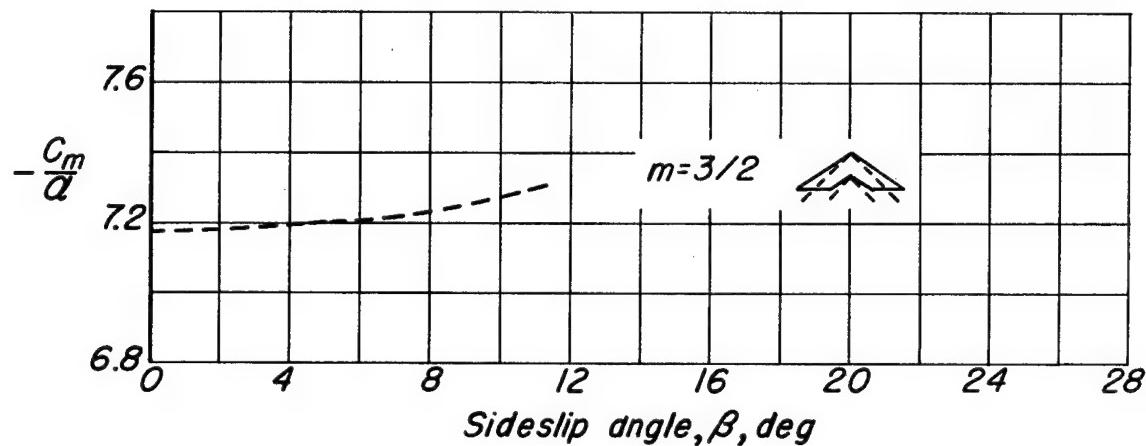
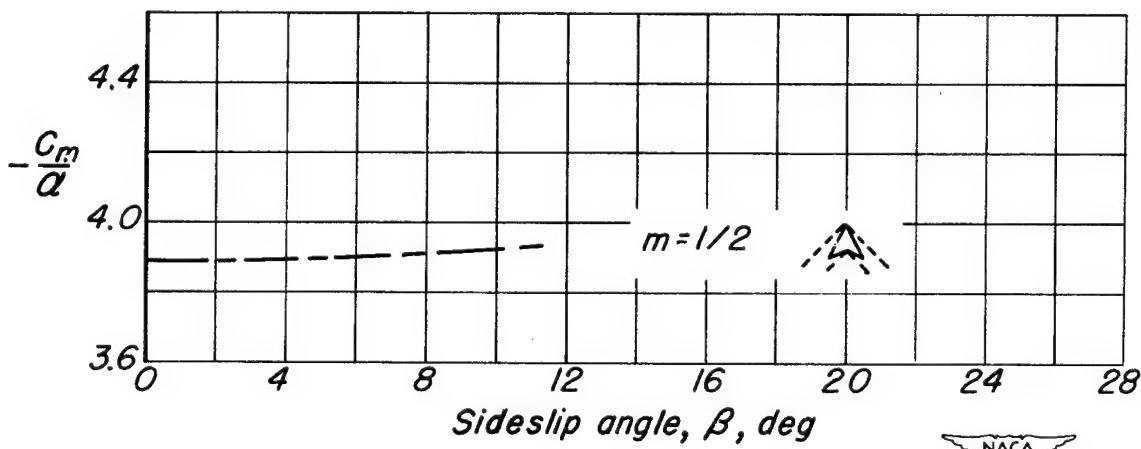
(c) Swept-back plan forms; $Bm \geq 1$.(d) Swept-back plan forms; $Bm < 1$.

Figure 5.- Concluded.

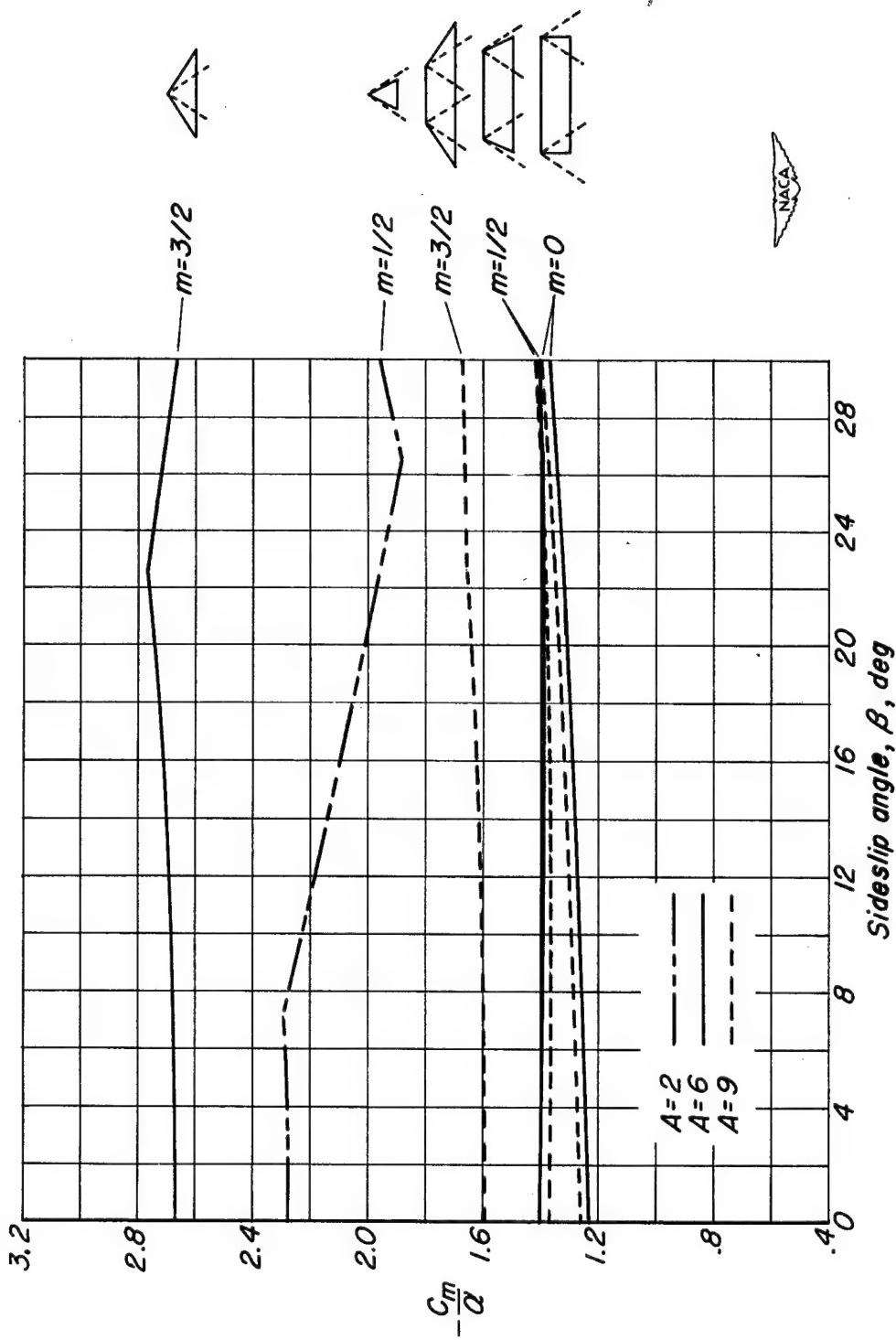
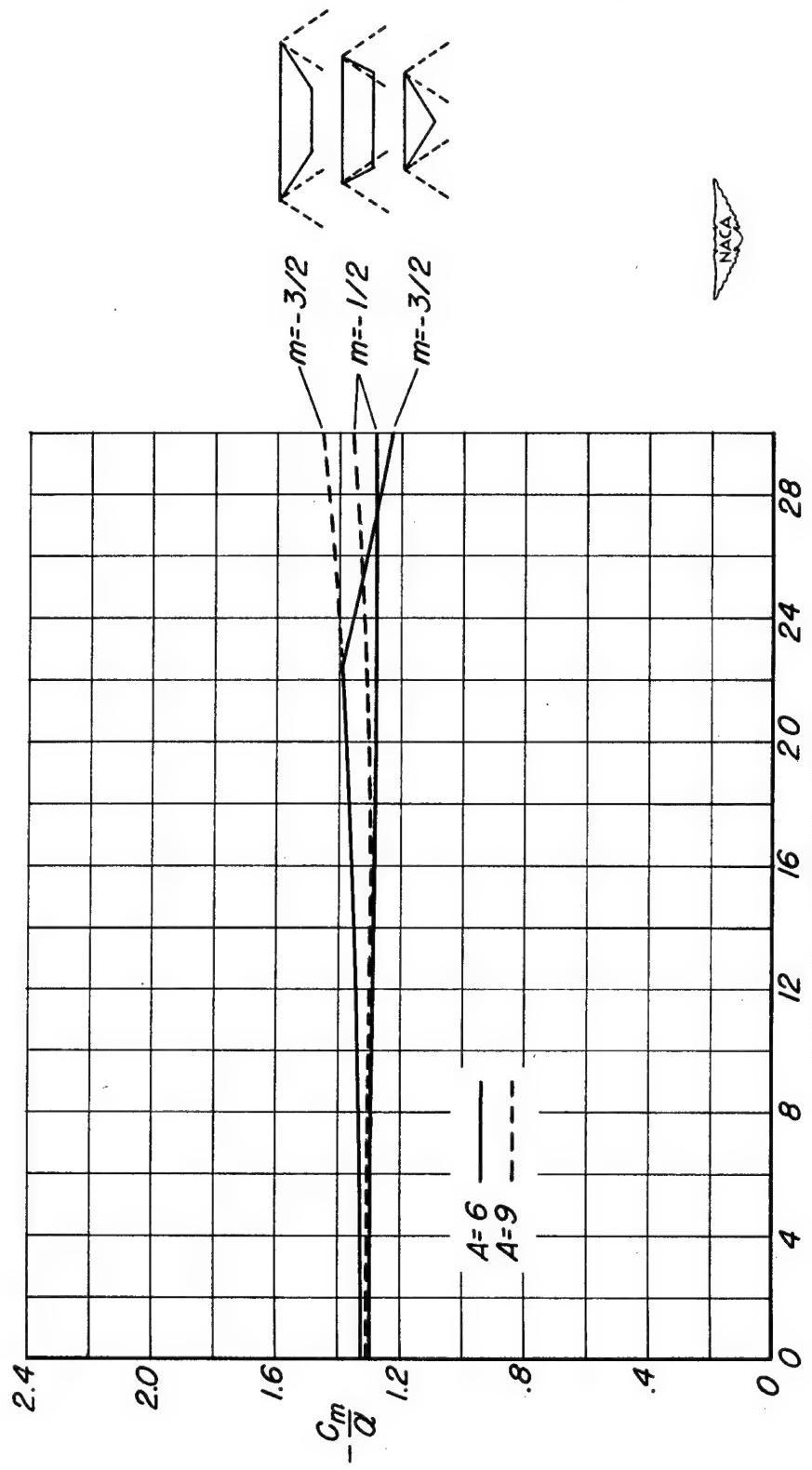


Figure 6.—Variation of pitching-moment coefficient per unit angle of attack with sideslip angle for $B=1.5$.

(a) Triangular, trapezoidal, and rectangular plan forms; $m \geq 0$.



(b) Triangular and trapezoidal plan forms; $m < 0$.

Figure 6.—Concluded.

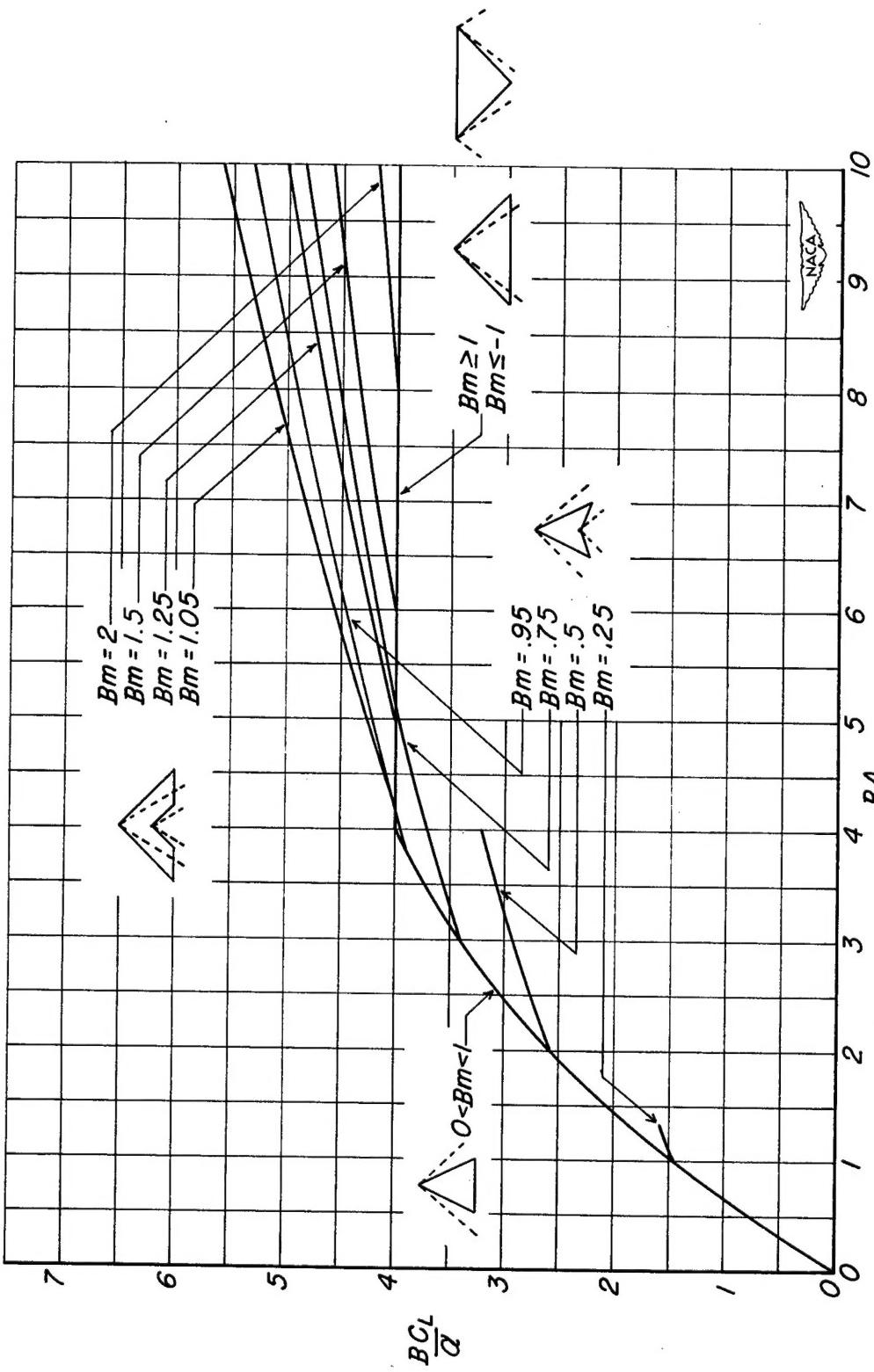
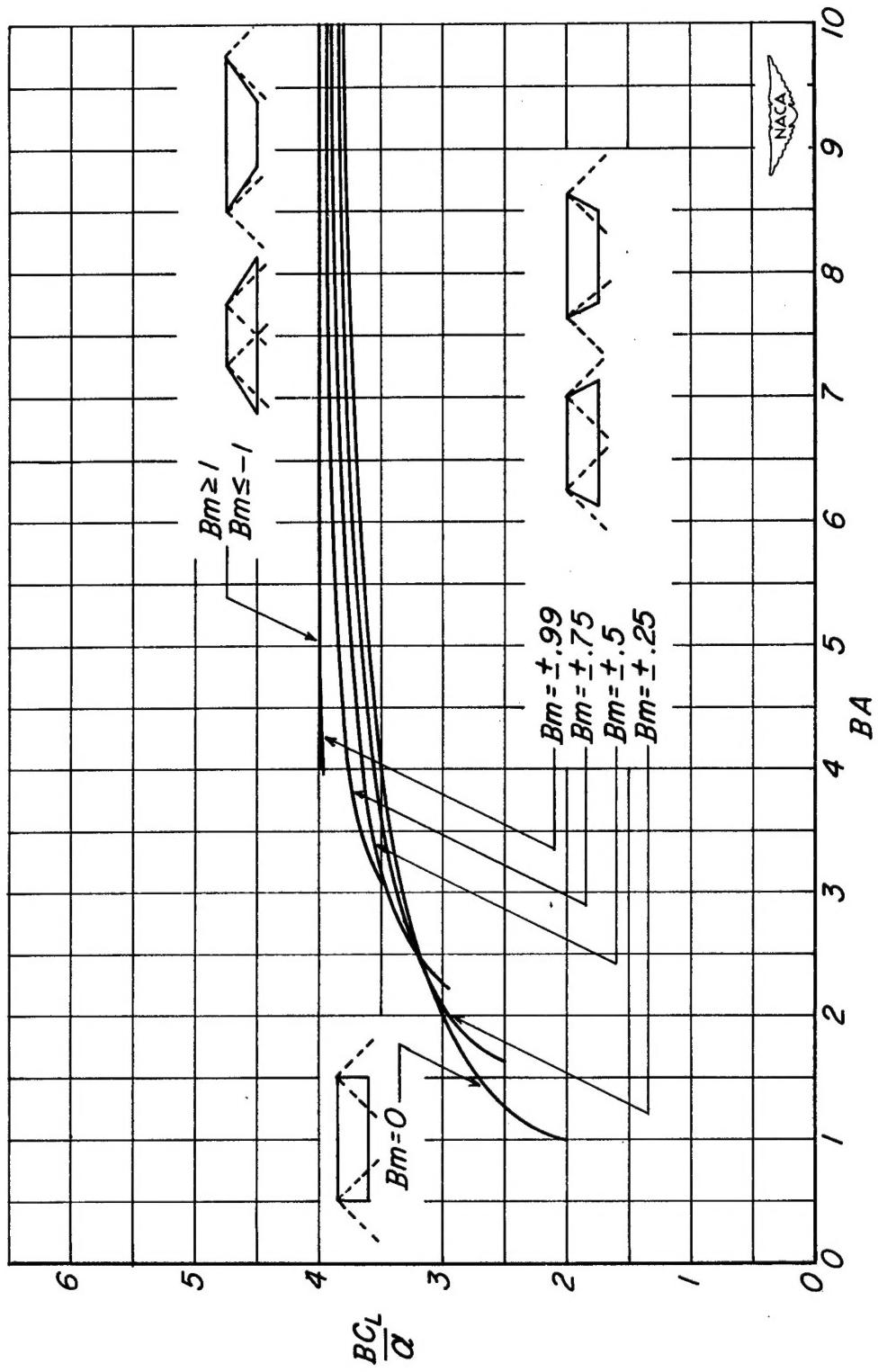


Figure 7.- Variation of modified lift coefficient BC_L per unit angle of attack with aspect ratio parameter Bm at zero sideslip.
 (a) Triangular and swept-back plan forms.



(b) Trapezoidal and rectangular plan forms.

Figure 7 - Concluded.

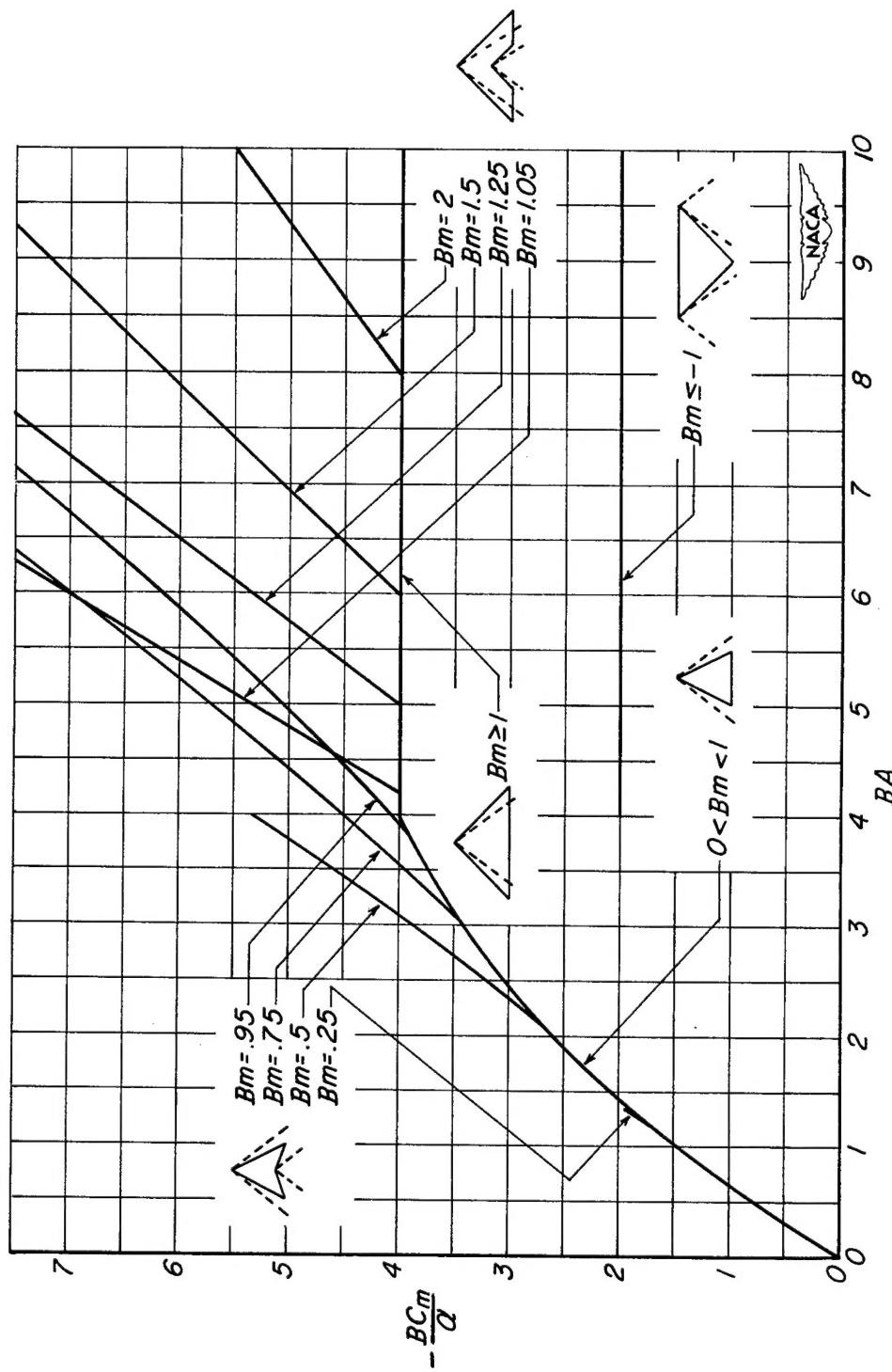
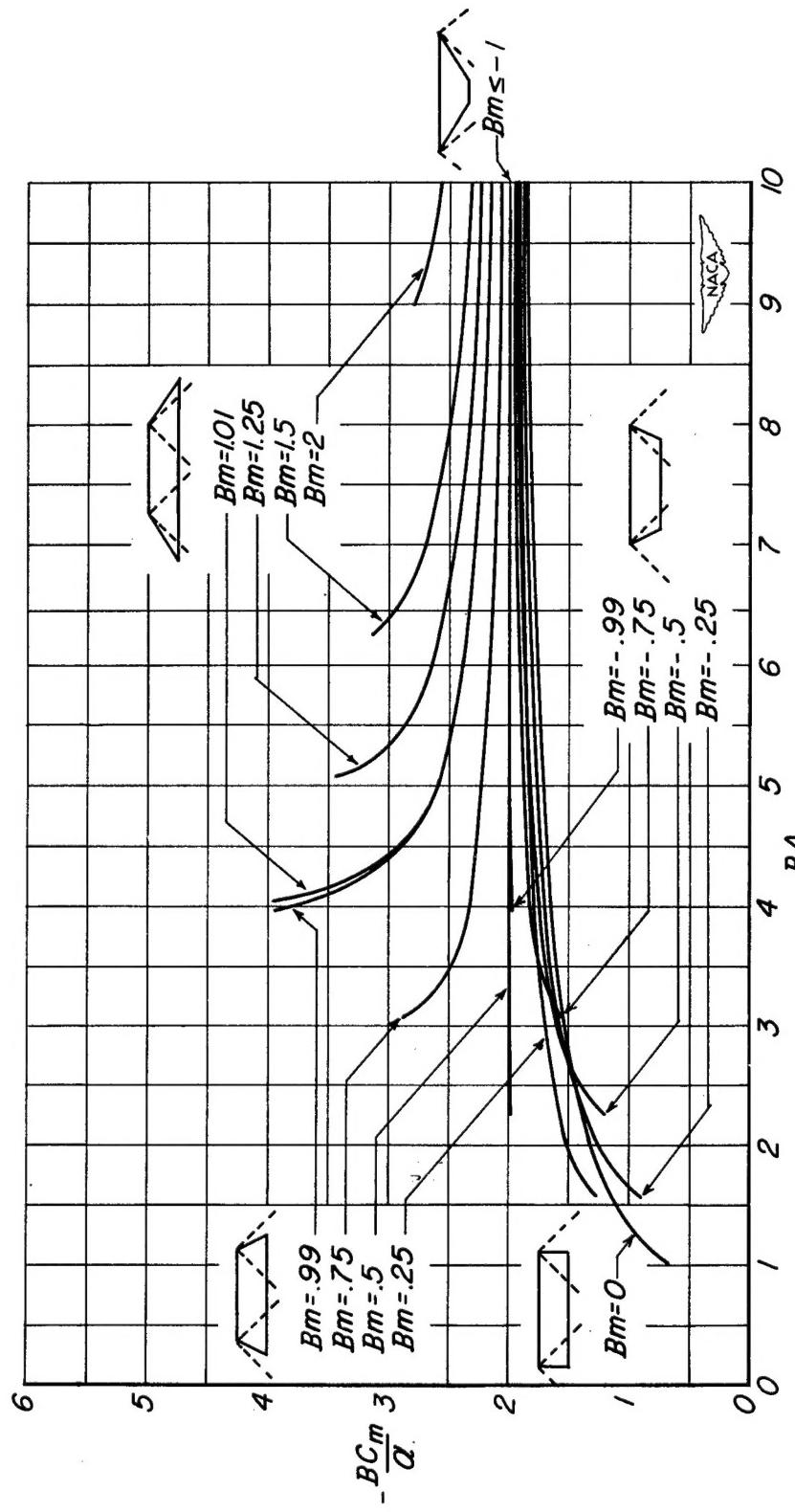


Figure 8.—Variation of modified pitching-moment coefficient BC_m per unit angle of attack with aspect ratio parameter BA at zero sideslip.
(a) Triangular and swept-back plan forms.



(b) Trapezoidal and rectangular plan forms.

Figure 8. - Concluded.